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Testing for purchasing power parity: a nonlinear approach

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Testing for purchasing power parity: A nonlinear approach

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to my family

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CHAPTER 1. INTRODUCTION

The doctrine of Purchasing Power Parity (PPP) has been a cornerstone of many exchange rate determination models. Barro (1984) even considers PPP to be "the central theoretical proposition of international finance." In its absolute version, PPP simply states that the equilibrium exchange rate (domestic currency per unit of foreign currency) equals the ratio of domestic to foreign price levels. The relative version of PPP requires that percentage changes in exchange rates and relative prices be equal over time.

Despite its theoretical appeal, the empirical evidence about long-run PPP is quite mixed. People generally agree that PPP is not a short-run principle in the sense that price level movements do not begin to offset exchange rate swings on a monthly or even annual basis. i.e., short-run deviations from PPP can be easily observed. However, the validity of PPP in the long run remains controversial. Some people found that the real exchange rate follows a random walk, other people rejected the cointegration relationship between exchange rate and prices, these facts suggest that shocks have infinitely long-lived effects, and hence the deviation from PPP can never be eliminated over time. On the other hand, some researchers reported evidence in favor of long-run PPP, they claim the mild convergence toward PPP by saying that the half lives of deviations are about three to four years, which is equivalent to 2% monthly speed of adjustment.

Testing the PPP hypothesis is important since it is used, implicitly or explicitly, at least as a long-run relationship, in much of current international economics. For example, early monetary models of the exchange rate (see for example the studies in Frenkel and Johnson, 1978) assume continuous purchasing power parity, while sticky-price exchange rate models originally developed by Dornbusch (1976) allow the exchange rate to deviate from purchasing power parity in the short run, although it is retained as a long-run equilibrium condition. Another example regarding the importance of the validity of PPP is that we usually imply PPP when we compare the standard of living across countries. To see it, let's go further a little bit. First of all, the real meaningful index representing the standard of living should be real per-capita output. Let y and y* denote the real per-capita output in domestic country and foreign country respectively. Then if $y>y^*$, we can claim that the domestic country enjoys a higher standard of living than foreign country. However, in the real world, when people try to do this comparision, they simply compare per-capita income rather than real per-capita output. i.e., they compare Py and P^*y^*, given P and P^* are price levels in domestic country and foreign country respectively. But considering that Py and P^*y^* wind up with different currency units, people naturally take the product of exchange rate e (expressed as the units of domestic currency per unit of foreign currency) and P^*y^* to transform to the same currency measure. Equivalently stated, actually people think the domestic country has a higher standard of living than foreign country if and only if $Py>eP^*y^*$. Now we can clearly see that $y>y^*$ can only be infered from $Py>eP^*y^*$ if $P=eP^*$, which is exact absolute version of PPP. Hence, in summary, if PPP does not hold, these international comparisions can be quite misleading. e.g., as the work of Kravis and associates (1978,1982,1983) has shown, the real income of poor countries is severely underestimated when actual exchange rates are used to make the comparison. The low relative price of non-tradables in poor countries yields for poor countries true purchasing power of income significantly above what exchange rate-converted income suggests.

Also, when people claim that a specific currency is "overvalued" or "undervalued", this judgement is based on PPP. The currency is "overvalued" when its exchange rate makes domestic goods look expensive relative to similar goods sold abroad and "undervalued" in the opposite case. Thus, the domestic currency is "overvalued" if and only if $P>eP^*$.

The remainder of this dissertation is organized as follows: Chapter 2 is a careful literature review part, it starts with the basis of PPP and is followed by a detailed description of the evolution of the PPP tests as well as the results of these tests. Strength and weakness regarding each category test are discussed also. At the end of this chapter, we present some possible theoretical explanations about why deviations from PPP could arise.

Chapter 3 stresses on the theoretical explanation of the downward stickness of the price. We present four possible sources for asymmetric adjustments of price, in which the last one is a partial equilibrium menu cost model approach. The asymmetry of price is carefully discussed here to justify the application of asymmetric unit-root test and cointegration test in Chapter 5.

A detailed description of the prevailing testing methodology as well as the new developed asymmetric test methodology of Purchasing Power Parity is given in Chapter 4. It first covers conventional unit-root test and cointegration test, namely, Dickey-Fuller test, Engle-Granger test and Johansen test. Following that, is the rationale to consider about switching from symmetric scenario to asymmetric scenario. Finally, considering that the asymmetric test methodology is very new, we fully show in the paper the development of these tests, namely, asymmetric Enders-Granger unit-root test and asymmetric Enders-Siklos cointegration test. At the end of this chapter, is the power comparison between conventional test methodology and new asymmetric test methodology. This part is very essential in the sense that the major flaw of the conventional test is the low power to reject.

In Chapter 5, we move on to test for PPP by applying the new asymmetric unit-root test and cointegration test to a special characteristic data set. There are two improvements here: first, the substitution of asymmetric test method to symmetric test method catches the essential asymmetric adjustments of price and therefore has more power to reject the null hypothesis. this is very important in the sense that many people attribute the failure of PPP to the low power of conventional test to distinguish between unit-root process and near unit-root process. Secondly, to infer PPP from "law of one price", it is required that the national price index should be constructed by using the same mixture of goods and by using the same weights on each good across countries. However, in practical, people usually use CPI or WPI as national price index, these two indices do not satisfy these requirements. Due to the kindness of Mr.Pippenger, we are able to get a new data set which does meet these requirements. These two improvements together make the acceptance of long-run PPP more likely. Our RATS programming results support our expectations. We find strong evidence in favor of PPP. Detailed interpretaion of the results are given. Also at the end of this chapter, to have further insight into the dynamic adjustment process of prices and exchange rates, asymmmetric error-correction models are estimated and impulse response funtions are also depicted. Explanations are given by using Italy-UK case. At the very end of chapter five, we make some efforts to explain the empirical failure of PPP in Germany-US case and Germany-Canada case.

We do a fractional cointegration test comparison in Chapter 6 to demonstrate that the incorporation of the asymmetric test methodology is very necesssary to supporting PPP. A more powerful mean-reversion detecting method alone, even combined with a better data set, works not good enough. This results shows that it is very essential to be aware of the asymmetric adjustment mechanism and take it into prior consideration.

Chapter 7 is mainly about a Fourier convergence test of PPP. Fourier convergence test is very general in that it does not specify a specific alternative hypothesis. Hence it can be used as an approximation test for any nonlinear convergence, it is particularly appropriate for the case when people have little information concering the real structure. However, our results show that in testing PPP, this general method is not very much preferred to the conventional method, let alone the threshold model method. The rational is that we do have theoretical justification of applying asymmetric threshold method to testing PPP. Hence, the TAR and M-TAR model better captures the real adjustment mechanism and in turn performs very well.

Chapter 8 summarizes the findings.

CHAPTER 2. LITERATURE REVIEW

Basis of PPP

One of the first expositions of Purchasing Power Parity can be dated back to Cassel's (1922) idea that the nominal exchange rate should reflect the purchasing power of one currency against another. Cassel proposed that a purchasing power exchange rate exists and it is the rate toward which the nominal exchange rate would tend, in the absence of trade imbalances, speculation, central bank intervention, and other impediments to trade.

Simple PPP asserts that exchange rates should be equal to relative price levels in different countries. The basic motivation underlying PPP is rather straightforward: first, goods-market arbitrage ensures law of one price for each traded good, i.e.,

$$(2.1) \qquad p_t(i) = p_t^*(i) + s_t$$

where $p_t(i)$ is the log of the time-t domestic-currency price of good i., $p^*(i)$ is the analogous foreign-currency price, and s_t is the log of the time-t domestic-currency price of foreign exchange.

Law of one price basically says that absent from tariffs and transportation costs, free trade in goods should ensure identical prices across countries. If the law of one price holds for every individual good, then it follows naturally that it must hold for any identical basket of goods.

Most emperical tests, however, do not attempt to compare identical baskets, but use CPIs and WPIs instead. Generally, this will lead to the validity problem due to different weights and mixes of goods.

Ever since the initiation of the idea of PPP, a large amount of empirical tests has been done. These studies feature different sample periods, different currencies, different specifications, and different estimation methods.

Sample periods frequently employed are the 1920s (a period of hyperinflation) and post-Bretton Woods flexible exchange rates regime that began in 1973. Occasionlly people use fixed exchange rate era or the earlier period of the gold standard. Some studies span various regimes.

The currencies investigated also differ, though the majority of studies focus on bilateral rates of the industrialized countries against the US dollar. Sometimes bilateral rates between two countries other than the United States are cross computed from the data set, whereas others explicitly select a bilateral rate excluding the US. More recently, purchasing power parity has been tested for developing countries.

The specifications used to test purchasing power parity also varies depending on whether the trivariate relationship between the exchange rate, the domestic price series, and the foreign price series; the bivariate relationship between the exchange rate and the domestic-foreign price ratio; or the univariate real exchange rate is being examined. The trivariate relationship is the most general, it imposes neither symmetry (price coefficients of the same magnitude but opposite sign) nor proportionality (price coefficients restricted to be [1,-1]). The bivariate specification implicitly imposes symmetry, and the univariate specification imposes both symmetry and proportionality.

The specifications may further differ according to the price series used; WPI and CPI are two often choices. It's worth noting that constructing of other price indices are possible.

Finally, various methods have been employed to test purchasing power parity, especially since the mid-1980s, when significant progress on econometrical methods was made. Recent studies use Dickey-Fuller and augmented Dickey-Fuller tests of the real exchange rate; Perron-type tests of the real exchange rate allowing for a one-time structural break; variance ratio tests; the Engle-Granger two-step method; error correction models; the maximum likelihood estimation procedure by Johansen; and, most recently, fractional integration methods.

Following Froot and Rogoff (1995), three different stages of PPP tests can be distinguished depending on different test methodology. Specifically, they are:

(1) stage-one tests in which the null hypothesis is that PPP holds

(2) stage-two tests in which the null hypothesis is that the real exchange rate is a random walk.

(3) stage-three tests in which the null hypothesis is that deviations away from any linear combination of prices and exchange rates is permanent

In the following section, a detailed description of the evolving of testing methodology is given. Then the results for each testing methodology as applied to PPP will be examined.

Three Stages of PPP Tests

Stage One: Least Square Estimation on Testing Simple PPP

The special feature of stage-one test is centering on coefficient restrictions. One example of this type of test is Frenkel (1978) paper. Frenkel ran regressions of the form

$$s_t = \alpha + \beta(p_t - p_t^*) + \epsilon_t \qquad (2.2)$$

for a number of hyperinflationary economies. where the exchange rate s and the domestic and foreign price series p and p^* are expressed in logarithms. α and β are estimated regression coefficients, ϵ is an error term.

Frenkel found estimates of β quite close to one during 1920s hyperinflation period while far from one during 1970s for industrialized countries. It is worth stressing that Frenkel was not so much interested in the properties of the error term, as in whether the slope coefficient was one.

The major flaw in the stage-one tests was the failure to take explicitly into account the possible nonstationarity of relative prices and exchange rates. Briefly, classical hypothesis testing of $\beta=1$ is inappropriate when the regressors are nonstationary because their variance do not converge to a constant. Reported standard errors will thus be underestimated.

Another obvious problem with stage-one test is that exchange rates and prices are simultaneously determined, and there is no compelling reason to put exchange rates on the left-hand side, rather than visa-versa.

Stage Two: The Real Exchange Rate as a Random Walk

In stage-two tests, the null hypothesis is that the real exchange rate follows a random walk, with the alternative hypothesis being that PPP holds in the long run. These tests distinguish themselves with stage-one tests in that they impose- rather than estimate- the hypothesis that $\beta = 1$, and test - rather than impose - the hypothesis that the log of the real exchange rate

$$(2.3) \qquad r_t = s_t - p_t + p_t^*$$

is stationary. Early stage-two testing job can be found from Darby (1983), Adler and Lehman (1983), Hakkio (1984), Huizinga (1987) and Meese and Rogoff (1988).

The main problem with stage-two tests is low power. Now it is widely acknowledged that PPP is not a short-run relationship, price level movements do not begin to offset exchange rate swings on a monthly or even annual basis. i.e., if convergence to PPP does exist, this process is rather slow, unfortunately, the prevailing stage-two test methodology (e.g., Dickey-Fuller test) has little power to distinguish between a random-walk real exchange rate and a stationary real exchange rate that reverts very slowly. This problem turns out to be even serious when applying highly volatile floating exchange rates, because the noise can easily mask slow convergence toward long-run equilibrium.

Worrying about the insufficient power, much of the evolution of stage-two testing focused on finding longer or broader data sets, and implementing more powerful unit-root tests. Such effort can be found from Abuaf and Jorian (1990), Kim (1990). Of course, these efforts usually incur the problem of regime change.

In the following subsections, major stage-two testing methodologies are described.

Dickey-Fuller and Augmented Dickey-Fuller Tests

The Dickey-Fuller (1979) and augmented Dickey-Fuller (1981) tests for a unit root, can be adapted to test for nonstationarity in the real exchange rate r_t, defined as ($s_t - p_t + p_t^*$).

The test specification is:

$$(2.4) \qquad \Delta r_t = \alpha + \theta r_{t-1} + \sum_i \eta_i \, \Delta r_{t-1-i} + v_t$$

where t is a time trend. Lags of Δr_t may be necessary to ensure that v_t is white noise. The null hypothesis is that $\theta = 0$, which implies that the real exchange rate contains a unit root.

If the null hypothesis of $\theta = 0$ cannot be rejected, the real exchange rate is considered a random walk. Random walk behavior means that the real exchange rate is the outcome of a sequence of real shocks, each of which permanently alters the level of the real exchange rate. i.e., there is no tendency for the real exchange rate to return to its mean or trend. Consequently, purchasing power parity cannot hold.

Equation (2.4) can also be used to calculate Phillips (1987) Z test, which allows for conditional heteroskedasticity of the residual.

Perron-Type Tests

Perron (1989) criticizes unit root tests based on the fact that a series that is stationary about a trend with a one-time structural break mimics the behavior of a unit root series. Thus, the power of unit root tests to distinguish between alternative hypotheses may be low.

Perron recommended detrending a series with allowance for a level shift and/or change in trend. The researcher may impose the date for the breakpoint based on a priori information, or may estimate the breakpoint as in Perron and Vogelsang (1992). Dummy variables are introduced to the Dickey-Fuller or augmented Dickey-Fuller test to capture these changes.

Application of this method on PPP can be found from Perron and Vogelsang (1992) and Flynn and Boucher (1993).

Variance Ratio Tests

Cochrane (1988) first introduced variance ratio tests for GNP. He exploit the fact that the variance of the first difference of a unit-root series grows proportionally with the time separating two observations. For example, $var(y_{t-1}-y_t) = \sigma^2$ whereas $var(y_{t+k} - y_t) = k \cdot \sigma^2$. Thus, for a unit root series, $var(y_{t+k} - y_t) / k \cdot var(y_{t+1} - y_t) = 1$ for any $k= 1,\ldots\ldots, T$. On the other hand, for a stationary white noise series, the ratio approaches zero as k approaches infinity. Thus, the behavior of the variance ratio reveals information about the stochastic behavior of a series. Like the Dickey-Fuller test and the Perron-type tests, the variance ratio tests are typically conducted for the real exchange rate.

Examples of variance ratio tests of PPP are Glen (1992) and Grilli and Kaminsky (1991).

Fractional Integration Tests

Fractional integration tests entertain a broader range of alternative hypotheses to the unit root hypothesis. A fractionally integrated process allows the real exchange rate to evolve according to:

$$\text{(2.5)} \qquad \Phi(L)(1-L)^d r_t = \chi(L)\epsilon_t$$

where $\Phi(L)$ and $\chi(L)$ are polynomial lag operators with roots outside the unit circle and ϵ_t is white noise. If the parameter $d = 0$, then the real exchange rate is confined to a class of stationary ARMA processes described by $\Phi(L)$ and $\chi(L)$. If $d=1$ and $\Phi(L) = \chi(L) = 1$, then the real exchange rate follows a random walk. The strength of this class of processes is that it allows for fractional integration, i.e., $0<d<1$. A series that is integrated of order d where $0<d<1$ is fractionally integrated. Fractionally integrated series exhibit reversion to a mean, but at a much slower rate than a stationary series, encompassing them under the alternative hypothesis may enhance one's chances of rejecting the random walk null.

Cheung and Lai (1993b) find that the tests tend to have more power than augmented Dickey-Fuller tests in detecting mean reversion, particularly for $0.35<d<0.65$. Fractional integration methods rely on spectral analysis and have not yet been widely employed.

Stage Three: Cointegration Tests

Cointegration techniques are designed to test if there exists a long-run equilibrium relationship among a few nonstationary variables. It can be used to test weak version of PPP, since they require only that *some* linear combination of exchange rates and prices be stationary. To put it in another way, instead of testing the real exchange rate $r_t = s_t - p_t + p_t^*$ is stationary as in stage-two test, cointegration test asks only whether

$$(2.6) \qquad s_t - \mu p_t + \mu^* p_t^*$$

is stationary for *any* constant μ and μ^*.

Easy to see, stage-two test imposes both the symmetry and proportionality restrictions by setting $\mu = \mu^* = 1$. Hence, it may simply be thought of as the univariate case.

Any incremental power from stage-three tests over stage-two tests must therefore come from relaxing the symmetry and proportionality restrictions. If only proportionality restriction is relaxed, it is a bivariate cointegration test, which examines the long-run equilibrium relationship between the exchange rate and the price ratio. If both restrictions are relaxed, then we get the most general trivariate cointegration tests.

We describe the major cointegartion test methodologies below.

The Engle-Granger Test for Cointegration

Engle-Granger tests of cointegration follows a two-step procedure. The test applies either the standard or the augmented Dickey-Fuller test for a unit root to the residual from the first-stage "cointegrating regression" of the levels of the variables. For purchasing power parity studies where each of $[s_t, p_t, p_t^*]$ series is nonstationary, the cointegrating regression, using the logarithms of the exchange rate and domestic and foreign price series, is:

$$(2.7) \qquad s_t = \alpha + \beta_1 p_t + \beta_2 p_t^* + \mu_t$$

Depending on the specification used, β_1 may or may not be set equal to $-\beta_2$. Cheung and Lai (1993a) pointed out that the imposition of symmetry may affect conclusions about cointegration. They favor the least restrictive version for testing long-run purchasing power parity.

The augmented Dickey-Fuller test for cointegration is:

$$(2.8) \qquad \Delta\mu_t = \theta\mu_{t-1} + \sum_i \eta_i \, \Delta\mu_{t-1-i} + \epsilon_t$$

These tests examine whether $\theta = 0$, which is equivalent to testing for a unit root in μ_t. If the hypothesis that $\theta = 0$ cannot be rejected, the exchange rate and the domestic and foreign price series are not cointegrated. The existence of a long-run relationship would be rejected. Engle and Yoo (1987) provide the critical values. Some studies report the closely related Philips-Perron Z statistics, which take account of possible autocorrelation and heteroskedasticity.

Johansen Test for Cointegartion

Johansen (1988) method is a maximum likelihood procedure. An inability to reject the null hypothesis of zero cointegrating vectors means the absence of cointegration. It improves Engle-Granger test in that it is a one-step full-information maximum likelihood approach, it does not require choosing a single right-hand side variable, hence the parameter estimates are more efficient, and the Johansen test for cointegration is therefore more powerful than a two-step Engle-Granger test.

Table 2.1 summarizes about twenty-one papers.

Table 2.1: Characteristics of Studies of Purchasing Power Parity[a]

	Abuaf and Jorion	Adler and Lehmann	Ardeni and Lubian	Cheung and Lai (1993a)
Sample Period				
Fixed parities		*		
Flexible exchange rates	*	*		*
1920s			*	
Pre-World War II	*	*		
Specifications				
$[s, p, p^*]$			*	*
$[s, (p-p^*)]$ or $[p, s-p^*]$				
$[s-p+p^*]$	*	*		
Price Index				
Consumer price index	*	*	?	*
Wholesale price index		*	?	*
Methods				
OLS or GLS				
Engle-Granger			*	
Error correction				
Johansen				*
Dickey-Fuller	*	*		
One-time break				
Variance ratio				
Fractional Integration				
Results				
Symmetry and proportionality?	imposed	imposed	found in 7 of 10 cases	rejected in most cases
Stationary relationship between s, p and p^*?	Yes	No	No	Yes

[a] * means the paper as indicated in the column heading uses the sample period, or specification, or method, or price index as indicated by the row heading.

Table 2.1: (Continued)

	Cheung and Lai (1993b)	Choudry et al.	Corbae and Ouliaris	Davuytan and Pippenger
Sample Period				
Fixed parities	*			
Flexible exchange rates	*	*	*	*
1920s				*
Pre-World War II	*			
Specifications				
$[s,p,p^*]$				*
$[s,(p-p^*)]$ or $[p, s\cdot p^*]$	*	*	*	
$[s-p+p^*]$		*		*
Price Index				
Consumer price index	*	*	*	*
Wholesale price index		*		
Methods				
OLS or GLS				*
Engle-Granger		*		
Error correction			*	
Johansen				
Dickey-Fuller		*		
One-time break				
Variance ratio				
Fractional Integration	*			
Results				
Symmetry and proportionality?	symmetry imposed	imposed	imposed	symmetry imposed
Stationary relationship between s, p and p^*?	Yes, but not for subsamples	Yes	No	Yes

Table 2.1: (Continued)

	Diebold et al.	Edison and Klovland	Enders	Flynn and Boucher
Sample Period				
Fixed parities		*	*	*
Flexible exchange rates			*	*
1920s		*		
Pre-World War II	*			
Specifications				
$[s, p, p^*]$				
$[s, (p-p^*)]$ or $[p, s \cdot p^*]$		*	*	*
$[s-p+p^*]$	*			*
Price Index				
Consumer price index	*	*		*
Wholesale price index	*		*	*
Methods				
OLS or GLS		*		
Engle-Granger		*	*	*
Error correction		*		
Johansen				
Dickey-Fuller			*	*
One-time break				*
Variance ratio				
Fractional Integration	*			
Results				
Symmetry and proportionality?	imposed	symmetry imposed; proportionality rejected.	symmetry imposed	imposed
Stationary relationship between s, p and p^*?	Yes	Yes	No, except for US-Japan	No

Table 2.1:(Continued)

	Frenkel	Glen	Grilli and Kaminsky	Kim
Sample Period				
Fixed parities			*	*
Flexible exchange rates	*	*	*	*
1920s	*			
Pre-World War II		*	*	*
Specifications				
$[s, p, p^*]$				
$[s, (p-p^*)]$ or $[p, s \cdot p^*]$	*			*
$[s-p+p^*]$		*	*	*
Price Index				
Consumer price index	*	*		*
Wholesale price index	*	*	*	*
Methods				
OLS or GLS	*			
Engle-Granger				*
Error correction				*
Johansen				
Dickey-Fuller			*	*
One-time break				
Variance ratio		*	*	
Fractional Integration				
Results				
Symmetry and proportionality?	symmetry imposed	imposed	imposed	imposed
Stationary relationship between s, p and p^* ?	Yes, but only for 1920s	Yes	Yes, but not always for Post-World War II period	Yes, in most cases, but not in the flexible rate period

Table 2.1: (Continued)

	Kugler and Lenz	Liu	McNown and Wallace	Patel
Sample Period				
Fixed parities				
Flexible exchange rates	*	*	*	*
1920s				
Pre-World War II				
Specifications				
$[s,p,p^*]$	*	*		*
$[s,(p-p^*)]$ or $[p, s\cdot p^*]$			*	
$[s-p+p^*]$			*	
Price Index				
Consumer price index	*	*	*	
Wholesale price index		*	*	*
Methods				
OLS or GLS				
Engle-Granger		*	*	*
Error correction			*	
Johansen	*	*		
Dickey-Fuller			*	
One-time break				
Variance ratio				
Fractional Integration				
Results				
Symmetry and proportionality?	found for 6 of 15 cases	found in fewer than half the cases	symmetry imposed; proportionality found in 3 of 4 cases	not tested
Stationary relationship between s, p and p^*?	Yes, in 10 of 15 cases for DM-base currencies	Yes, in 6 of 9 cases in Latin America	Yes, for high inflation economies with WPI	No, except for 3 of 15 currencies

Table 2.1: (Continued)

	Perron and Vogelsang	Pippenger	Taylor and McMahon
Sample Period			
Fixed parities			
Flexible exchange rates		*	
1920s			*
Pre-World War II	*		
Specifications			
[s,p,p*]		*	
[s,(p-p*)] or [p, s-p*]		*	*
[s-p+p*]	*		*
Price Index			
Consumer price index	*		
Wholesale price index		*	*
Methods			
OLS or GLS			
Engle-Granger		*	*
Error correction			*
Johansen			
Dickey-Fuller		*	*
One-time break	*		
Variance ratio			
Fractional Integration			
Results			
Symmetry and proportionality?	imposed	not tested	not tested
Stationary relationship between s, p and p* ?	Yes, with allowance for level shift in mean	Yes, for 7 of 9 Swiss franc-based currencies	Yes, in most cases except for pound sterling

Results of Testing Purchasing Power Parity

The major results of stage-one tests are PPP holds for hyperinflationary economies while strong rejections of PPP can be found outside of hyperinflations. However, due to the obvious problem of test methodology, this conclusion does not receive much attention. In the following subsections, we'll focus on introducing the main empirical results of stage-two and stage-three tests.

Results of Stage-Two Tests

Results for Post-Bretton Woods Data

The basic fact in the empirical literature is that if one applies unit-root tests to bilateral industrialized country monthly data, it is difficult to reject the null of a unit root for currencies that float against each other.

For currency pairs that are fixed, the results are mixed. In Mark's (1990) tests for the 1973-1988 period, the intra-European exchange rate come closest to rejecting a random walk, although it is only for the Belgium/Germany currency pair that a random walk null can be rejected at the 5% confidence level. Chowdhury and Sdogati (1993) look at the 1979-1990 period, when the EMS was in place, they strongly reject the random walk for bilateral rates of various European currencies against the Deutsche mark, but not for European exchange rates against the U.S. dollar.

Early stage-two tests usually employed post-1973 data, due to the short time span which is available and the fact that the reversion to PPP, if it does exists, tend to be rather slow, people are very concerned about the fact that stage-two tests lack sufficient power to reject. The solutions to this problem come either from expanding data set or from employing more powerful test methodology or both.

Actually, much of the evolution of stage-two tests stressed on expanding data set. To see how important the issue of power is, and to gain a sense of how much data is needed to reject plausible alternatives, we can refer to Frankel (1986, 1990), according to his autoregression results, 72 years of monthly data are needed to reject random walk null if the true half-life of PPP deviations is 36 months (3 years). Obviously, with a longer half life, even more data would be required. Therefore, the Post-Bretton Woods sample period is far too short to reliably reject the random walk hypothesis.

Two approaches to dealing with the power problem from the data perspective have been tried in the past: one is to look at a number of currencies simultaneously, and the other is to look at long-horizon data sets encompassing both pre- and post-Bretton Woods data.

Tests Using Cross Section Data

Easy to see, if four currencies are simultaneously used, then 18 years of data would be sufficient to reject the null hypothesis given the half life of PPP deviation is 36 months. (since $18 \cdot 4 = 72$). Hakkio (1984) firstly used cross-section data to gain power. he employed GLS to allow for cross exchange rate correlation in the residuals in four exchange rates against the dollar. In spite of the enhanced power of his test, Hakkio was unable to reject the random walk model.

Abuaf and Jorion (1990) did a similar job. Currencies of 10 industrialized countries are studied over the period 1973-1987. The US dollar is the base currency. Mild rejection of random walk hypothesis was found due to gained power.

The most positive evidence in favor of PPP might be Cumby (1993) 's interesting "Hamburger" study, by using 7 years (1987-1993) of the dollar price of McDonald's Big Mac in up to 25 countries, Cumby found that the large cross-section yields enough power to detect reversion toward the law of one price. Namely, only 30% of the deviation in one year pass to the next.

Tests Using Longer Time Span

The second way to improve power is to extend the sample period. e.g., Frankel (1986) applies 116 years (1869-1984) of data for the dollar/pound real exchange rate; Edison (1987) estimated error-correction mechanism by using dollar/pound data for the years 1890-1978; Johnson employed 120 years of Canadian dollar/U.S. dollar exchange rate data; Abuaf and Jorion (1990) use time series data from 1901-1972 for eight currencies; Glen (1992) uses variance ratios to test for mean reversion in the real exchange rate for 9 bilateral exchange rates over the 1900-87 period.

The common results of the above studies are statistically significant convergence toward PPP. The half life of PPP deviation ranges from 3 - 7 years.

It is worth noting here that the long-sample studies all combine low variance pre-Bretton Woods exchange rate data with the highly volatile post-Bretton Woods data. Because the convergence appears easier to detect in fixed rate than in floating rate, these papers actually did not answer the question of whether mean reversion would be detected in 100 years of floating rate data, for example. Lothian and Taylor (1994) attempted to cast some light on this issue. By using a simple Chow-test to compare first-order AR coefficients before and after Bretton Woods, they find that the null of no structural change cannot be rejected. Hence, they conclude that no evidence supports the view that the inclusion of fixed-rate periods biases unit roots tests of the real exchange rate.

Results of Stage-Three Tests

Lots of people have applied cointegration approach to test for PPP. A partial list includes Edison and Klovland (1987), Corbae and Ouliaris (1988), Enders (1988), Kim (1990), Mark (1990), Fisher and Park (1991), Cheung and Lai (1993a), and Kugler and Lenz (1993). Surveys include Giovannetti (1992) and Breuer (1994).

Generally, cointegration tests are more successful than stage-two tests in rejecting the random walk hypothesis. Within the cointegration tests, rejections of the no-cointegration null occur more frequently for trivariate systems than for bivariate systems. Hence, one can conclude that weakening the proportionality and symmetry restrictions makes the residuals appear more stationary.

To get more detailed inference of cointegration tests, let's give a description of two typical papers in the following:

Cheung and Lai (1993a)

Cheung and Lai (1993a) use the Johansen method for the trivariate system $[s_t, p_t, p_t^*]$. The method imposes neither the symmetry nor the proportionality restriction. For the sake of comparison, Cheung and Lai redo the estimation for the bivariate system $[s_t, (p_t-p_t^*)]$. They also perform augmented Dickey-Fuller tests for a unit root in the real exchange rate. A comparison of the methods illustrates the dramatic change in results when the symmetry and/or the proportionality restriction is imposed. Their data cover 1974-89 and five industrialized country currencies, with the US dollars as the base currency. They report results for both the consumer price index and the wholesale price index.

Cheung and Lai report evidence of cointegration for all five currencies. They reject the hypothesis of zero cointegrating vectors. In some cases, they reject the hypothesis of one cointegrating in favor of two, which means that the exchange rate and the domestic and foreign price series are pairwise cointegrated. The coefficient estimates of β_1 and β_2 -- as in equation (7) -- vary from a low of 1.035 to 25.422 when the consumer price index is used, and from 0.35 to 11.414 when the wholesale price index is used. In all cases, except for the dollar-pound rate using WPI, proportionality is rejected. In four out of five cases symmetry is rejected when the CPI is used and in three out of five cases when the WPI is used.

Cheung and Lai compare these results with results using bivariate and univariate specifications. For the univariate real exchange rate, which imposes the symmetry and proportionality, they find that unit root behavior cannot be rejected for the real exchange rate, regardless of the price index used. Results from the bivariate specifications are more similar to those from the trivariate specifications.

Kim (1990)

Kim (1990) compares several different tests for long-run purchasing power parity. Using the Phillips-Perron Z statistics, he tests for a unit root in the real exchange rates of five currencies of industrialized countries. He also uses the Engle-Granger test for a unit root in the cointegrating regression residual from a bivariate specification. Finally, he estimates an error correction model and tests for the significance of the equilibrium error term as a way of detecting long-run purchasing power parity. He compares results for the consumer price index with those from the wholesale price index. His data roughly cover 1900-87. He also examines subperiods.

Kim finds the real exchange rate to be stationary when the wholesale price index is used for all five currencies. However, when the consumer price index is used instead, only the French franc-dollar rate is stationary. Thus, cointegration is detected more frequently when the wholesale price index is used at least with the univariate specification. The Engle-Granger method for the bivariate specification is also used. In four of five cases cointegration is confirmed using the wholesale price index. Results from the error correction model find the same thing. The Engle-Granger method detects cointegration in only two of five cases when the consumer price index is used.

How Could Deviations from PPP Arise?

General Explanation

Since the law of one price is the central building block of PPP, any factor leading to the failure of this law could lead to the failure of PPP as well. Hence the first straightforward reason for deviations from PPP is the presence of nontradables or "home" goods. We know that arbitrage keeps relative prices between tradables equal across countries, but this breaks down for relative prices between nontradables as well as between tradables and nontradables. Also, it is worth noting here that even highly "traded" goods embody substantial nontraded inputs. For example, the retail price of bananas includes not only the traded goods inputs, but local shipping, rent and overhead for the retailer, and labor. As a matter of fact, for many seemingly highly-traded goods, these indirect costs can far outweight direct traded-goods costs. Consequently, the deviations from law of one price not only arise from nontradable goods but also could arise from traded goods.

To see it more clearly, suppose the real exchange rate is $q_t \equiv s_t - p_t + p_t^*$ and the price index in each country is a weighted average of traded and nontraded goods prices. i.e.,

$$(2.9) \qquad p_t = \gamma p_t^T + (1-\gamma)p_t^N$$

$$(2.10) \qquad p_t^* = \gamma^* p_t^{*T} + (1-\gamma^*)p_t^{*N}$$

Then, we can write,

$$(2.11) \qquad q_t = (s_t - p_t^T + p_t^{*T}) - (\gamma-1)(p_t^T - p_t^N) + (\gamma^*-1)(p_t^{*T} - p_t^{*N})$$

so real exchange rate depends on deviations from the law of one price in traded goods, as well as on the relative price of traded and nontraded goods within each country. Think about the case $\gamma=\gamma^*$, then the sum of the second and third term on the right hand side equals zero if and only if $p_t^T - p_t^N = p_t^{*T} - p_t^{*N}$. But we know from the first paragraph of this section it cannot be true. Therefore, the existence of nontradables contribute to the deviation from PPP. Moreover, the first term on the right hand side represents the part of the role of tradable goods. Rogers and Jenkis (1995) find that on average, 81 percent of the variance in the real CPI exchange rate is explained by changes in the relative price of traded goods. Equivalently stating, the deviations from the law of one price in traded goods dominate real exchange rate fluctuations. Because CPI is a retail price index, this supports the view that retail goods generally contain substantial nontraded components.

In addition to the arbitrage perspective explanation, structural model can also be given to explain the deviation from PPP. This kind of model focuses on more fundamental factors such as productivity, government spending, and strategic pricing decisions by firms. Among the many structural models, the most famous one is Balassa (1964)-Samuelson model, in which they claim that after adjusting for exchange rates, CPIs in rich countries will be higher relative to those in poor countries, and that CPIs in fast-growing countries will rise relative to CPIs in slow-growing countries.

The main logic of Balassa-Samuelson model is as follows: first of all, technological progress has historically been faster in the traded goods sector than in the nontraded goods sector. Moreover, this traded-goods productivity bias is more pronounced in rich countries. As a consequence, CPI levels tend to be higher in wealthy countries. Why? increased productivity in the traded-goods sector increases wages in that industry and hence raises economy-wide wages, but without accompanying productivity gains (or only with a relative smaller productivity gain) in the nontraded-goods sector, costs and prices there must rise and hence the growing country's relative price of nontraded goods will rise. We know the CPI weights more on nontraded goods, therefore, CPI will go up. Because the productivity growth bias is more pronounced in rich countries, we can claim that CPI will increase more in rich countries than in poor countries.

Another category of the models which can explain the deviations from PPP are called "Asset Approach", e.g., Dornbusch (1976) model. An essential feature of these models is that commodity markets adjust slowly relative to asset markets. As a result, commodities are nontradable in the short run and tradable in the long-run. This model explains short-run deviation through the "overshooting" of exchange rate, but implies long-run deviation should be eliminated gradually.

Also, as we will discuss in detail in Chapter 5, unequal weights in the price indices introduces measurement error and is a potential reason for the deviation from PPP.

Dornbusch (1980) Model Approach about PPP Deviations Due to Productivity Gain

Let's consider about a two-country, infinity-good model in which labor is the only factor of production. Constant return to scale is assumed so that for each industry, there is a constant unit labor requirement. Across countries technology differ, i.e., unit labor requirements differ. To deal with the many-commodity case, in Figure 2.1 we choose as our horizontal axis the unit interval (0,1) and we assume that to each point on the interval there corresponds a particular commodity. Let z be an index that ranges from zero to one. Then we can index commodoties by z.

Then we denote a(z) as the unit labor requirement of commodity z in the home country and a*(z) abroad. The relative unit labor requirement is defined as

$$(2.12) \qquad A(z) \equiv a^*(z)/a(z)$$

which is a measure of the relative technical efficiency at home and abroad.

We depict A(z) schedule in the fashion of decreasing relative efficiency. i.e., we place toward the origin the commodities for which the home country has relatively low unit labor requirement, and toward the unity, we place those for which the foreign country is relatively more efficient. Suppose the equilibrium domestic and foreign wage rates are W and W* respectively. Then from Figure 2.1, we know that to the left of the dividing point z_0 (where $A(z_0)$

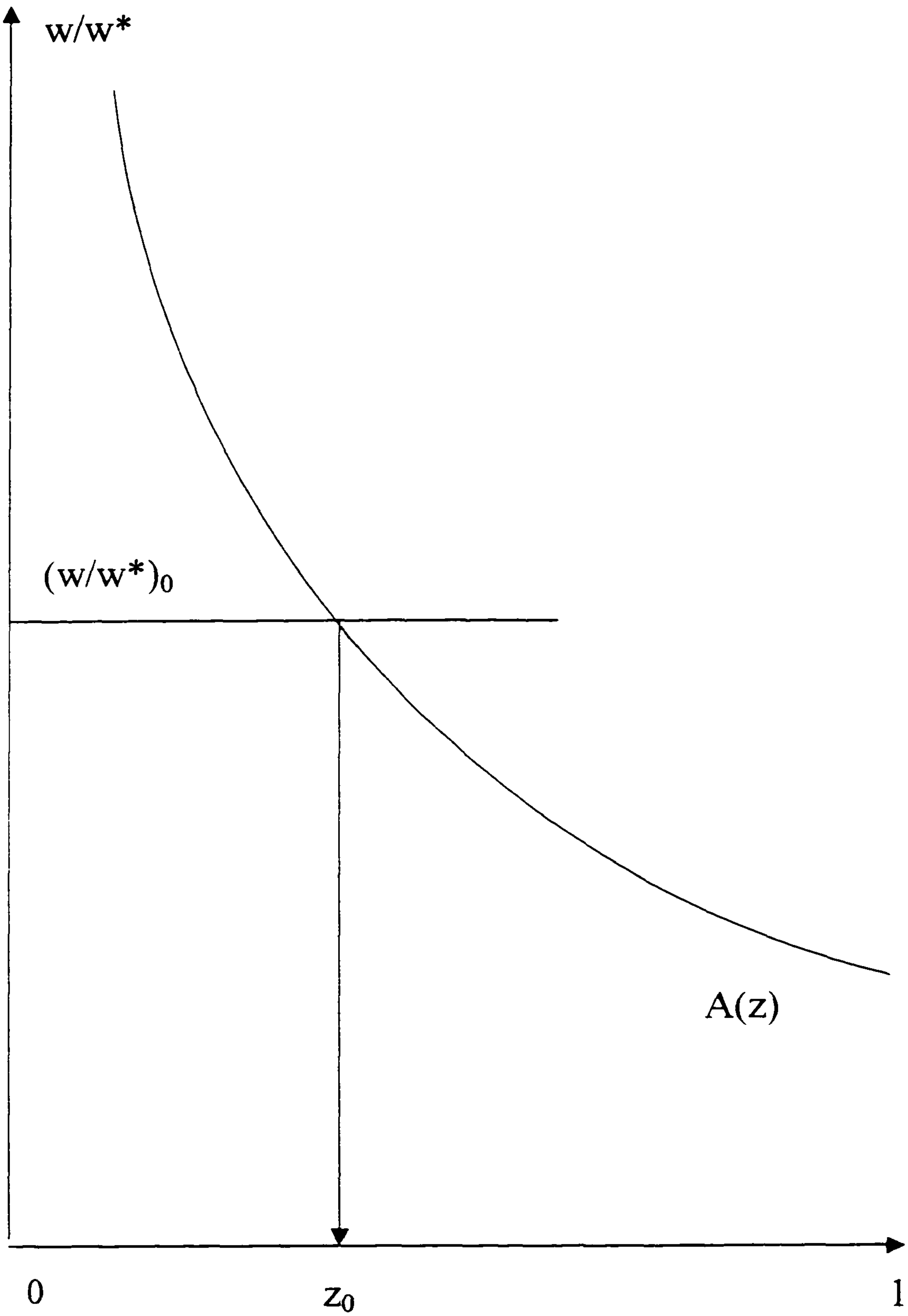

Figure 2.1. Dornbusch (1980) Model about PPP Deviation

= W/W*), are those commodities produced by home country. while to the right of z_0, are those produced by foreign country. It's easy to see,

$$(2.13) \qquad z_0 = \phi(W/W^*) \qquad \phi' < 0.$$

It can be easily infered from (2.13) that with a higher relative wage the home country is less cost competitive, thus producing a smaller range of goods.

Assume the price level is an weighted index of the three sets of prices - importables, exportables, and nontraded goods:

$$(2.14) \qquad P = P_m^{(k-\theta)} P_x^{\theta} P_n^{(1-k)} \qquad P^* = P_m^{(k-\theta)} P_x^{\theta} P_n^{*(1-k)}$$

where P_i is a price index of all the commodities in a particular group. In comparing the two countries's indexes we note immediately that they face the same traded goods prices and the law of one price holds for the traded goods. From (2.14), relative price level can be expressed as:

$$(2.15) \qquad P/P^* = (P_n/ P_n^*)^{1-k}$$

For simplicity we can imagine a single home good with a unit labor requirement c per unit of output and c* abroad. With prices equal to unit costs we then have:

$$(2.16) \qquad P/P^* = (cW/c^*W^*)^{1-k} \equiv \beta(W/W^*)^{1-k}$$

where $\beta \equiv (c/c^*)^{1-k}$ represents the relative level of labor productivity in the home goods sector. (2.16) shows that relative price levels are determined by labor productivity in the home goods sector and by relative wages.

We now turn to the role of productivity change in the traded goods sector of home country and investigate their impact on relative price levels. Inspection of Figure 2.1 shows that the growth in domestic productivity in the traded goods sector shifts the A(z) schedule upward as domestic labor requirements are reduced. For a given relative wage that implies an increase in the range of goods produced at home and the demand for goods and labor at home goes up. With an excess demand for domestic goods at the initial equilibrium we now have an adjustment in relative wages. Our relative wage rises, thereby partially offsetting the gain in cost competitiveness.

The rise in the home country's equilibrium relative wage implies by (2.16) that our price level rises relative to that abroad. The reason is that the increased labor productivity in the traded goods sector leads to a rise in money wages and therefore to a rise in costs and prices of nontraded goods where productivity has not changed. As relative price levels are determined by home goods prices, the relative price level of the country experiencing productivity growth must increase.

Balassa (1964) has shown that the theoretical argument has empirical support. Taking the GNP deflator as a comprehensive price index, including both nontraded and traded goods, he showed that countries with high productivity growth experience a rise in the GNP deflator.

"Pricing To Market" Theory

It has been noticed very early that similar or even identical goods can be sold at significantly different prices in different markets. This difference is too large to be explained by transportation costs and/or other trade barriers. One famous example is automobile. e.g., in the early 1990s, a Nissan automobile built at the Japanese company's Sunderland plant in northeast England could be bought from a dealer near the plant for £16,215. The same model sold in Japan for only £13,375--- despite the cost to Nissan of shipping the car 10600 miles from Sunderland to Tokyo. It is also strongly remarked that the prices of many imports into the United States did not fall to the degree that one might expect given the strong dollar of the early 1980s. The prices of European luxury cars even rose in US dollar terms despite huge depreciation in European currencies against the US dollar. Since prices in Europe, in European currencies, did not rise dramatically, the effect was to create large differences between prices of the same automobiles in the United States and Europe, so large that it actually gave rise to "gray markets" in which individuals and firms bypassed normal distribution channels to import automobiles directly from Europe.

According to Krugman (1987, p 49), "the phenomenon of foreign firms maintaining or even increasing their export prices to the United States when the dollar rises may be described as 'pricing to market' (PTM) ". It's easy to see, PTM is an example of the deviation from law of one price in traded goods. If it does play an important role in bilateral trade, then PPP could fail between these two countries. A number of studies, including Knetter (1989), find that PTM is more pronounced for Germany and Japanese exporters than it is for American exporters. Moreover, Krugman (1987) looked at Germany-US trade pattern and claimed that PTM can be easily detected in the industry of machinery and transport equipment, which is a very large trading sector between the two countries. This fact serves as a very nice reason for the empirical failure of PPP between US and Germany as shown later in this dissertation.

In the following we give three model explanations due to Krugman (1987) about PTM phenomenon. The first two are static models in the sense that the belief on the part of firms that the dollar's rise is temporary does not play any role in their pricing behavior. The last one is the dynamic model in that in the model it is assumed to be crucial that the dollar's rise is taken to be temporary.

Supply and Demand

Imagine a two-country world, namely, U.S. and EC, two currencies, dollar and ecu. Let P be the dollar price of some U.S. importable, P* the ecu price, and e the number of ecus per dollar. Also, let S(P), S*(P*) be the supply from each region, while D(P) and D*(P*) are the demands. Then equilibrium may be described by:

$$(2.17) \quad S(P) + S^*(P^*) - D(P) - D^*(P^*) = 0$$

In addition, we assume law of one price:

$$(2.18) \quad P^* = eP$$

Equilibrium assuming law of one price and the effects of a dollar appreciation may be illustrated as in Figure 2.2. Clearly a dollar appreciation will lead to the fall of dollar price and the rise of the ecu price. Thus, the dollar price does not fall in full proportion to the appreciation. Hence, this model can explain why the dollar prices of BMWs fail to fall in proportion to the dollar's rise, but it cannot explain why these prices have fallen in Europe relative to the United States (i.e., a fall in P* relative to eP).

The key observance is by asserting the law of one price, the possibility of price divergence between the U.S. and EC is ruled out. To modify this model, we assume that there were an upward-sloping supply curve for transportation of importables to the U.S. market. Then the law of one price would be replaced with a new relation of the form:

$$(2.19) \quad P^* = eP - t$$

where t is marginal transport cost, and is increasing in the volume of U.S. imports:

$$(2.20) \quad t = t(D-S)$$

For the new model, a rise in the dollar would be accompanied by a fall in the U.S. price, and thus

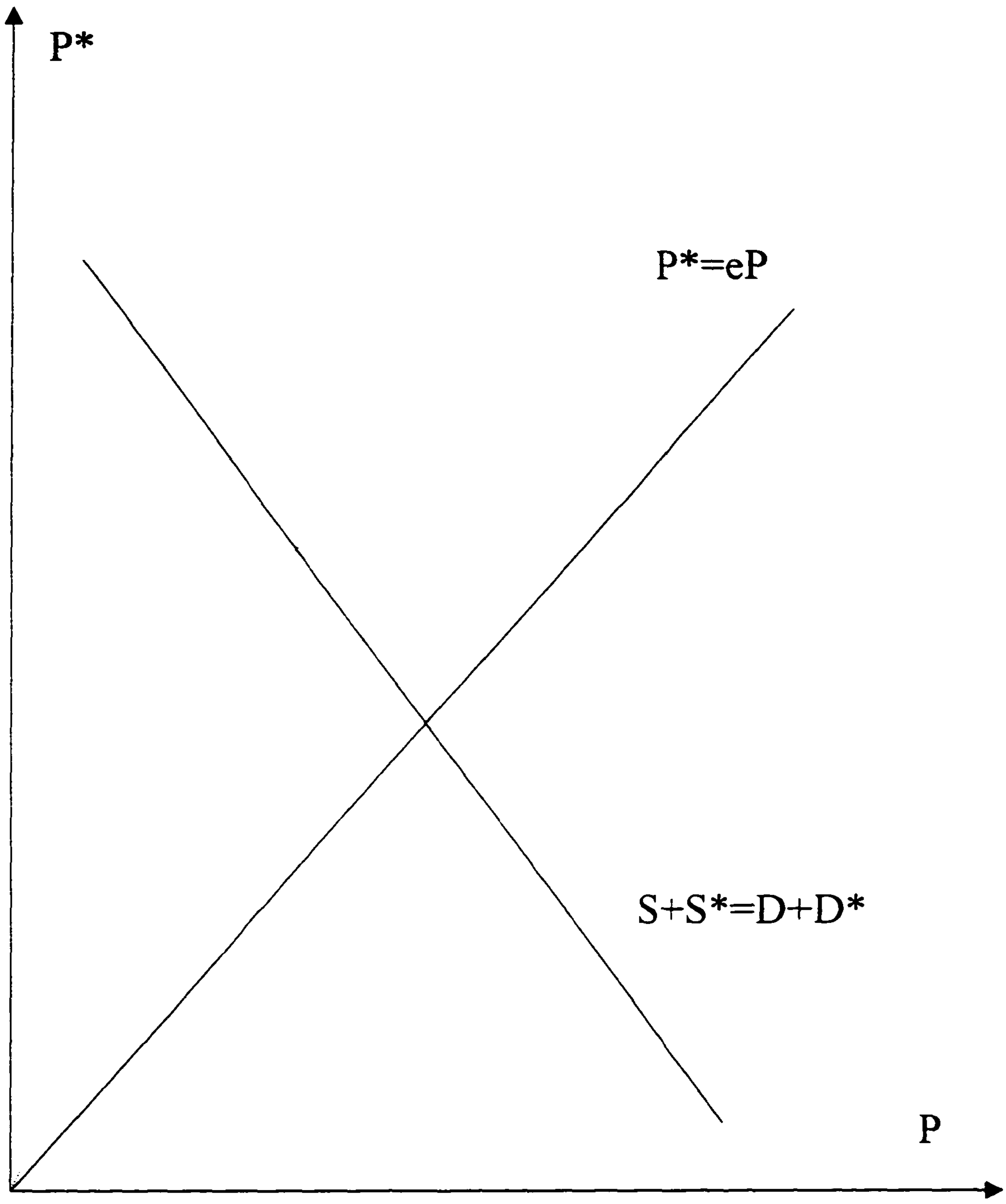

Figure 2.2. Krugman Supply and Demand Model about Pricing to Market

a rise in U.S. imports. The rise in imports would be associated with a rise in marginal transportation costs, and thus with a widened gap between U.S. and EC prices.

Then there comes the question about how plausible is it to suppose that marginal transportation costs are strongly upward sloping. In this sense, this model has flaw.

Monopolistic Price Discrimination

Let's assume a monopolistic firm that can sell either in the United States or the EC and that it has a constant marginal cost in ecus. Transport costs will be ignored. Then the monopolist's optimal pricing rule is:

$$(2.21) \quad P^* = c^*E^*/(E^* - 1)$$

$$(2.22) \quad eP = c^*E/(E - 1)$$

where c^* is marginal cost in ecus, and E and E^* are the elasticity of market demand in the United States and EC, respectively. E and E^* may of course depend on P and P^*.

From (2.21), P^* is invariant to e. The question is whether a rise in e will produce a more or less than proportional change in P.

Clearly, if the demand curve has constant elasticity, the U.S. price will fall in full proportion to the exchange rate change. In order to get pricing to market, we must have a fall in the elasticity of demand. that is, the elasticity of demand must be increasing in the price (so that it falls as the price falls).

In principle, price-discriminating monopoly can explain PTM if demand curves have the right shape. For luxury goods, we know, the elasticity of demand tends to be an increasing function of price.

Dynamic Model

In this model we ask whether slow adjustment of demand to the market price will give rise to a slow adjustment of the price itself. Suppose that there are lags in the effect of price on demand. Then a fim's pricing decision will in effect have a trade-off between low profits now and higher sales later.

Consider a foreign firm that plans to sell a good in the United States over two periods. In the first period it faces a demand $D_1(P_1)$, in the second a demand $D_2(P_1, P_2)$, Marginal costs are c^*/e_1, c^*/e_2. The firm will seek to maximize

$$(2.23) \quad (e_1P_1 - c^*)\, D_1(P_1) + R(e_2P_2 - c^*)\, D_2(P_1, P_2)$$

where R is a discount factor.

The essential question is whether P_1 will fall more if the exchange rate rises in both periods than it rises only in the first period. That is, will an exchange rate appreciation that is regarded as temporary have less effect on the price than one that is regarded as permanent? The answer to this question lies on how the second-period appreciation affects the incentive of the firm to keep its first-period price down. Note that the derivative of second-period profits with respect to P_1 is:

$$(2.24) \quad (dX_2/\, dP_1)(e_2P_2 - c^*) < 0$$

where X_2 is second-period sales, this may be rewritten as:

$$(2.25) \quad [(dX_2/\, dP_1)(P_1/X_2)][(e_2P_2 - c^*)X_2]/P_1$$

The first term in square brackets here is the cross elasticity of demand; the second term is second-period profits. The conclusion is that if expression (2.25) increases in absolute value in e_2, there will be an increased incentive to hold down the first-period price.

CHAPTER 3. ASYMMETRIC ADJUSTMENTS OF PRICE

Classical economists predict that markets are continuously cleared by flexible prices with instantaneous adjustment mechanism to external shocks. Keynesian theories, in contrast, claim sluggish price responses. Both of academic schools found empirical evidence. Namely, inflationary episodes support classical standing point while price adjustment behavior during contractions are more in favor of Keynesian prediction.

It seems that the conclusions of both sides can be reconciled into one single model with asymmetries in price adjustments. i.e., maybe they all describe price behavior correctly, but in different stages of business cycles. This claim can only be made if price respond asymmetrically to positive and negative deviations from the trend. To be specific, prompt producer corrections of prices that are below trends can account for the essential classical feature of flexible price responses. Conversely, reluctance to margin reductions for prices that are above trends can support the Keynesian description of slow adjustment that leaves prices at excessively high levels.

First of all, we define the *positive asymmetry of price* as the fact that prices are more readily to be raised in response to positive shocks than to be lowered in response to negative shocks. Then, the natural follow-up question is to detect the possible sources of positive asymmetries in price adjustment. In the following subsections, a few theories about the rationals regarding the positive asymmetry of price are given.

Theory One: Strategic Pricing by Oligopolistic Competitors

In their "trigger price" model of oligopolistic coordination, Green and Porter (1984) pointed out that in response to a negative cost shock, a firm might choose to maintain a prior price until demand conditions force a change.

Borenstein, Cameron and Gilbert (1997) furtherly investigate the responsiveness of retail gasoline price to crude oil price change, they find that in the retail gasoline market each firm chooses its selling price with imperfect information about the prices charged by others. Firms may choose to maintain prices above competitive (Nash) levels in response to negative cost shocks as long as their sales remain above a threshold level. A significant drop in sales would indicate price cutting by rival firms and would justify a price deduction as an optimal competitive response.

On the other hand, a significant positive crude price shock would trigger retail price increases, otherwise, retail margins would become negative given the typical thin margins in gasoline distributions. However, retail prices need not respond immediately to a negative crude price shock. The above observations together can explain the asymmetric pricing behavior by retail gasoline outlets. Of course, over time, random shocks in demand would lead the retailers to cut their prices in response to the threat of price cutting by rival firms.

While appealing, this theory has dificiencies. It explains how retailers may sustain prices above competitive levels, but does not explain how retailers will coordinate on a particular price. Actually there are multiple equilibria with prices above the competitive level. The three authors claim that the price that firms charged before a shock lowering wholesale prices is a natural focal point for coordination. Hence, an oligopolistic coordination equilibrium of the kind described here is consistent with a rapid response of prices to positive cost shocks and a slow response to negative shocks.

To summary, the response to cost shocks would be asymmetric because retailers would refrain from cutting prices in response to a negative shock and would instead rely on prevailing prices as a focal point for oligopolistic coordination. Retailers would not exercise similar restraint after a positive cost shock because they will lose money if they did not raise prices immediately after a significant positive cost shock.

The theory is sufficiently general to describe the price adjustment of other commidities.

Theory Two: Instrument Uncertainty

Greenwald and Stiglitz (1989) show that the producers tend to rely more on quantity adjustments than price adjustments over the business cycle by applying the analysis of Brainard (1967) on instrument uncertainty. The underlying assumptions are risk averse producers and price adjustments are perceived to be more risky than quantity adjustments.

Despite that instrument uncertainty was first used to interpret the (symmetric) inertia of producer prices, this theory can be modified to explain positive asymmetry in producer prices by adding an asymmetry due to costs of illiquidity. This altered theory has two essential building blocks: (i) the profit variance associated with price changes is significantly larger than that of output changes; and (ii) producer risk varies countercyclically due to imperfect credit markets.

Under the first condition, costs due to production change, such as worker hires or layoffs, are relatively predictable, and thus less risky. On the other hand, price changes have more uncertain impacts on profit due to unpredictable customer or competitor responses.

Under the second condition, in boom periods with large accumulations of retained earnings, the probability of exhausting internal liquid asset reserves is small, little attention is paid on the risk exposure associated with price variations and both price and output are varied to maximize the discounted stream of expected profits. However, in bad times, there is great concern about the uncertainty of projected receipts of cash flows. Producers become more cautious about changing prices to alter expected sales and respond to reduced demand by cutting planned production.

Theory Three: Wage Inertia

The downward stickiness of nominal wages will translate into the downward stickiness of producer price if the purchase of labor inputs is a major component of operating costs. Keynes (1936) suggests that employees may resist reduction in nominal wages because it is difficult for atomistic agents to coordinate their actions. Moreover, the efficiency wage hypothesis made by new-Keynesian economists suggests that the employers may also resist to lower wage level in fear that the loss of productivity might overpower the gain due to lower labor cost.

Theory Four: Menu Cost Model with Positive Trend Inflation

A Partial Equilibrium Approach

We consider a menu-cost model developed by Ball and Mankiw (1994) in which firms make regularly scheduled price changes, and by paying a menu cost, can also make special adjustments in response to shocks. In this model, asymmetries arise naturally with the addition of one feature:

positive trend inflation.

With trend inflation, positive shocks to firms' desired prices trigger greater adjustment than negative shocks of the same size. The intuition is that inflation causes firms' relative prices to decline automatically between adjustments. When a firm wants a lower relative price, it need not pay the menu cost, because inflation does much of the work. By contrast, a positive shock means that the firm's desired relative price rises while its actual relative price is falling, creating a large gap between desired and actual prices. As a result, positive shocks are more likely to induce price adjustment than are negative shocks, and the positive adjustments that occur are larger than the negative adjustments.

To see the above claims in detail, let's present in the following a partial equilibrium model in which trend inflation and costs of price adjustment produce asymmetric responses to shocks.

Assumptions and Specifications of the Model

Consider a single firm, let's define the following notations first:

θ: the firm's desired relative price in log.

p: aggregate price level in log.

π: steady inflation rate, which the firm takes as exogenous.

We can conclude from the above that the firm's desired nominal price is $p+\theta$, the price level in period t is $p_t = \pi t$, assuming $p_0 = 0$.

The firm adjusts the price as follows: every even period, after observing the current θ, the firm sets a single price for that period and the following odd period. However, it's not guaranteed that the firm will defend the price firmly. The firm can make an extra adjustment in an odd period by paying a menu cost C. usually the firm does so when there is a large shock to θ.

This specification combines time-contigent price adjustment (Ball et al. 1988) and state-contigent price adjustment (Caplin and Spulber, 1987). In accordance with the time-contigent model, the firm makes a regular schedule adjustment (every two periods); meanwhile, the firm also have the option to adjust in response to significant circumstances change, which provides the state-contigent model attributes.

It makes a lot of sense to combine these two polar cases in that price setting in actual economies has elements of both. Indeed, labor contracts are reviewed for fixed periods but can also be rewritten in special case.

The last important specification is about the firm's one-period loss funtion.

$$\text{(3.1)} \qquad \text{loss funtion} = (q-q^*)^2 + DC$$

where q is the firm's actual price, $q^* \equiv p+\theta$ is its desired price. D is a dummy that equals one if the firm pays the menu cost. The firm chooses even-period prices and whether to adjust in odd periods to minimize the average of its loss, with no discounting. If the firm does adjust in an odd period, it chooses the current q^*. Finally, let's impose the restriction that $C>\pi^2/4$.

A One-Time Shock

In this part we show the asymmetry in our model in the simple case of a one-time shock. We assume that, in period zero, the firm acts as if the probability of future shocks is zero. Whereas a shock occurs in period one.

We start by assuming that the desired relative price θ is zero in period zero and is expected to remain the same since the firm does not anticipate a shock. The firm's optimal nominal price,

$p+\theta$, equals zero in period zero and is expected to rise to π in period one. Given its quadratic loss, the firm chooses a price of $\pi/2$, the average of the two desired prices.

In period one, the shock happens: the firm's desired relative price changes to $\theta \neq 0$. To see how the firm may response, consider first its desired price adjustment. The fim's *ex post* optimal price is $\pi+\theta$, and its actual price entering the period is $\pi/2$. The desired adjustment is the difference between the two:

$$(3.2) \qquad \text{Desired adjustment} = \pi/2 + \theta$$

It is very important to note that the desired adjustment is asymmetric: the magnitude of the desired adjustment is larger for a positive θ than for a negative θ given $\pi/2$ is positive. The intuition is that, in the absense of shocks, inflation in period one pushes the firm's desired price q^* past its actual price q. A positive θ pushes q^* even further above q, creating a large desired adjustment. By contrast, a negative θ reduces q^* ; in this case, the shock offsets the need to catch up with inflation, and the firm desires a relatively small price changes.

This asymmetry in desired price adjustments leads to an asymmetry in actual adjustments. With a menu cost, a firm either chooses to make adjustment or not. If the firm does not adjust, the loss is $(q-q^*)^2 = (\pi/2 + \theta)^2$. If the firm does adjust, all the loss is menu cost C. Thus the firm fails to adjust if $C > (\pi/2 + \theta)^2$. Equivalent interval is:

$$(3.3) \qquad \theta \in [-\sqrt{C} - \pi/2, \sqrt{C} - \pi/2]$$

The first point to make is that the firm fails to adjust for a range of shocks. More important, this range is asymmetric: the lower bound is larger in absolute value than the upper bound. Relatively small positive shocks trigger adjustment, whereas prices are sticky for a larger range of negative shocks. Finally note that even if θ lies outside the range (3.3), so the firm adjusts, the price change is larger for a positive θ than for the corresponding negative θ. Both the asymmetry in the non-adjustment range and the asymmetry in the size of adjustments are demonstarted from this model.

A Distribution of Shocks

We now assume the firm faces a known distribution of shocks. Specifically, in period zero the desired relative price θ is zero; in period one it is drawn from a zero-mean, symmetric, and single-peaked distribution with cumulative distribution function $F(\theta)$. The knowledge that a shock may trigger adjustment in period one influences the price the firm sets in period zero. We solve jointly for the initial price, denoted x, and for the firm's price-adjustment rule.

Think of the behavior of the firm in period one. in period one, the firm takes x as given, since the firm's desired price is $\pi+\theta$, its desired adjustment is $(\pi+\theta-x)$. Similar to the previous appraoch, the firm fails to make the adjustment if $C>(\pi+\theta-x)^2$. That is:

$$\theta\in[\underline{\theta},\overline{\theta}]$$

where

$$(3.4) \qquad \underline{\theta}=x-\pi-\sqrt{C}; \quad \overline{\theta}=x-\pi+\sqrt{C}.$$

Notice that positive shocks induce adjustment more quickly than negative shocks as long as $x<\pi$.

Consider now behavior in period zero, the firm chooses the initial price x to minimizes:

$$(3.5)\qquad Loss = x^2 + \int_{\underline{\theta}}^{\overline{\theta}} (\pi+\theta-x)^2 dF(\theta) + [1-(F(\overline{\theta})-F(\underline{\theta}))]C$$

The first term in this expression is the firm's loss in period zero. the other terms are the expected loss in period one: the loss is $(\pi+\theta-x)^2$ if θ is in interval (3.4), and C if not in (3.4). The first order condition for minimizing (3.5) with respect to x leads to

$$(3.6)\qquad x = \frac{1}{1+F(\overline{\theta})-F(\underline{\theta})}[\pi(F(\overline{\theta})-F(\underline{\theta})) + \int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta)]$$

Using equation (3.4) and (3.6), one can show that

$$(3.7)\qquad 0<x<\pi/2$$

The fact that the initial price x is less than $\pi/2$ means that *ex post* price adjustment is even more asymmetric here than in the case of a one-time shock. This result is straightforward by comparing (3.4) and (3.3): the range of non-adjustment is shifted further to the left.

CHAPTER 4. TEST METHODOLOGY

Conventional Unit-Root Test and Cointegration Test

Dickey-Fuller Test

Unit-Root hypothesis test is widely used for inferring whether a time series $\{y_t\}$ follows a random walk or is stationary. It was first developed by Dickey and Fuller (1979,1981). To summarize briefly, consider the simple AR(1) model for $\{y_t\}$ with zero – mean, white noise innovations,

$$(4.1) \quad y_t = \gamma y_{t-1} + \varepsilon_t$$

It is easy to see that if γ is equal to unity, $\{y_t\}$ follows a random walk. If $-1<\gamma<1$, the sequence $\{y_t\}$ is stationary. To make equation (1) be more convenient for our purposes, reformulate it as:

$$(4.2) \quad \Delta y_t = \gamma^{*} y_{t-1} + \varepsilon_t$$

where $\gamma^{*} = \gamma - 1$. With this formulation, the Dickey-Fuller test for a unit-root is carried out by testing the hypothesis that γ^{*} equals zero.

Dickey and Fuller pointed out that the conventional t-test will tend, incorrectly, to reject the null hypothesis H_0: $\gamma^{*}=0$ due to biased estimator in the presence of unit root.The practical solution to this problem devised by Dickey and Fuller was to derive, through Monte Carlo methods, an appropriate set of critical values for testing the hypothesis that γ^{*} equals zero in an AR(1) regression when there truly is a unit root. i.e., the hypothesis may be carried out with a conventional t-test, but with a revised set set of critical values. A few of the critical values are reproduced in Table 4.1.

Table 4.1: Dickey-Fuller Critical Values

	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
Constant τ_μ								
50	-3.58	-3.22	-2.93	.-260	-0.40	-0.03	0.29	0.66
100	-3.51	-3.17	-2.89	-2.58	-0.42	-0.05	0.26	0.63

Moreover, it is worth noting that we also can incorporate constant and/or time trend into the AR model above, the corresponding critical values are also given in Table 4.1.

To account for serial correlation of errors, Dickey-Fuller test is carried out in the augmented model:

$$(4.3)\quad \Delta y_t = \gamma^* y_{t-1} + \Sigma\phi_j \Delta y_{t-j} + \varepsilon_t$$

This version is termed augmented Dickey-Fuller test.

Engle-Granger Test and Johansen Method

Let's denote differencing operator ∇=1-B, where B is the lag operator. If $\{\nabla^d X_t\}$ is stationary for some positive integer d but $\{\nabla^{d-1} X_t\}$ is nonstationary, we say that $\{X_t\}$ is integrated of order d, or $\{X_t\} \sim I(d)$.

If $\{\underline{X}_t\}$ is a k-variate time series, the I(d) process $\{\underline{X}_t\}$ is said to be cointegrated with cointegration vector $\underline{\alpha}$ if $\underline{\alpha}$ is a $k\times 1$ vector such that $\{\underline{\alpha}' \underline{X}_t\}$ is of order less than d.

People usually think about cointegration problem in the scenario of d=1. In this specific setting, cointegration test examines if there's a long-run equilibrium relationship among a few variables. It allows nonstationarity of variable itself but requires the stationarity of the linear combination of these variables. This property is often consistent with the reality. For instance, Many important economical variables are I(1) variables while there exists a long-run equilibrium relationship among them in that any deviation from this equilibrium cannot persist forever. The notion of cointegration captures the idea of univariate nonstationarity time series "moving together" and therefore has very wide applications.

Engle and Granger (1987) suggested a two-step approach to testing cointegrated process. The first step involves fitting the long-run relationship by least squares. As a second step, the hypothesis of no cointegration can then be tested by applying the Dickey-Fuller test to the residuals from the regression. If the residuals fail the test, the series are taken to be cointegrated. Otherwise, the specification would have to be reconsidered.

It is worth noting here that it is not possible to use the Dickey-Fuller Tables themselves, the problem is that the residual sequence is generated from a regression equation. The researcher does not know the actual error only the estimate of the error. Fortunately, Engle and Granger provide test statistics that can be used to test the hypothesis. If more than two variables appear in the equilibrium relationship, the appropriate Tables are provided by Engle and Yoo (1987). If you use n variables and a sample size of 100, the Engle and Yoo (1987) critical values for two through three variables at the 1%, 5% and 10% significance levels are shown in Table 4.2.

Table 4.2: The Engle-Yoo Critical Values

N	1%	5%	10%
2	-4.07	-3.37	-3.03
3	-4.45	-3.93	-3.59

In analogy to the augmented Dickey-Fuller test, assuming that the diagnostic checks indicate the problem of serial correlation, the following expression can be used:

$$(4.4) \qquad \Delta e_t = \alpha_1 e_{t-1} + \sum \alpha_{i+1} \Delta e_{t-i} + \varepsilon_t$$

where e_t is the estimate of the residual sequence from the least square regression of the long-run equilibrium.

Another popular cointegration test methodology is Johansen (1988) method. To state it briefly, consider equation (4.5):

$$(4.5) \qquad \Delta x_t = \pi x_{t-1} + v_t$$

where x_t and v_t are $(n \times 1)$ vectors; π is an $(n \times n)$ matrix of parameters.

The first step of Johansen method is to estimate π and to determine its rank. Equation (4.5) can take many different forms including the introduction of deterministic regressors, the addition of lagged changes in Δx_t. Note that the rank of π equals the number of cointegrating vectors. Hence, if we cannot reject the null of zero rank, we fail to reject the hypothesis of no cointegration.

Asymmetric Regime

Conventional unit-root test and cointegration test presuppose symmetric adjustment process. Note that in equation (4.2), γ^* is invariant with respect to y_t or Δy_t. In other words, the alternative hypothesis (i.e., $\gamma^* \neq 0$) implicitly assumes a symmetric adjustment process around $y_t = 0$ in that for any $y_t \neq 0$, Δy_{t+1} always equal $\gamma^* y_t$. Thus, $\gamma^* y_t$ can be viewed as an attractor whose pull is strictly proportional to the absolute value of y_t. Similar results hold in Engle-Granger test and Johansen test.

This implicit assumption of symmetric adjustment is quite problematic given we observe that many important economic variables display asymmetric adjustment paths. E.g., Neftci (1984) showed U.S. unemployment displays asymmetric adjustment over the course of the business cycle. Terasvirta and Anderson (1992) find that industrial production in 13 countries responds more sharply to negative shocks than to positive shocks. Similarly, Granger and Lee (1989) find that U.S. sales, production and inventories display asymmetric adjustment towards their long-run equilibrium relationship. Potter (1995) models changes in real U.S. GNP as a threshold adjustment process and Balke and Tomby (1992) show that various short-term interest rates exhibit threshold cointegration. Moreover, the intervention to economy by central bank and/or government usually is not necessarily symmetric. e.g., The Fed is more likely to take place when inflation rates are expected to rise than when they fall. Trade policy instruments are more often to be implemented when the country suffers a deficit than enjoys a surplus. Therefore, it seems to be appropriate to incorporate this asymmetric feature into the testing methodology.

Sichel (1993) discussed two variations of asymmetry: "Sharpness" (steepness) versus "deepness". Sharpness occurs when contractions are steeper than expansions and deepness occurs when troughs are more pronounced than peaks.

Corresponding to these two versions of asymmetry, by using the framework of Enders and Granger (1998), we introduce here two specific asymmetric models: Threshold Autoregressive (TAR) model and Momentum Threshold Autoregressive (M-TAR) model.

TAR model can be expressed as follows:

$$(4.6) \qquad \Delta y_t = \begin{cases} \rho_1 y_{t-1} + \varepsilon_t & if \quad y_{t-1} \geq 0 \\ \rho_2 y_{t-1} + \varepsilon_t & if \quad y_{t-1} < 0 \end{cases}$$

A necessary condition for the stationarity of $\{y_t\}$ is $-2<(\rho_1, \rho_2)<0$. Reformulate (4.6) as:

$$(4.7) \qquad \Delta y_t = \rho_1 I_t y_{t-1} + \rho_2 (1-I_t) y_{t-1} + \varepsilon_t$$

where I_t is the Heaviside indicator function such that

$$(4.8) \qquad I_t = \begin{cases} 1 & if \quad y_{t-1} \geq 0 \\ 0 & if \quad y_{t-1} < 0 \end{cases}$$

If the system is convergent, $y_t=0$ is the long-run equilibrium value of the sequence. If y_{t-1} is above its long-run equilibrium value, the adjustment is $\rho_1 y_{t-1}$ and if y_{t-1} is below long-run equilibrium, the adjustment is $\rho_2 y_{t-1}$. Since adjustment is symmetric if $\rho_1=\rho_2$, symmetric scenario is a special case of asymmetry.

Notice that the TAR model can capture aspects of "deep" movements in a sequence. If, for example, $-2<\rho_1<\rho_2<0$, the negative phase of the $\{y_t\}$ sequence will tend to be more persistent than the positive phase.

If we replace Heaviside indicator function in equation (4.8) by the following expression:

$$(4.9) \qquad I_t = \begin{cases} 1 & if \quad \Delta y_{t-1} \geq 0 \\ 0 & if \quad \Delta y_{t-1} < 0 \end{cases}$$

Equation (4.7) along with (4.9) are called M-TAR model.

M-TAR model can explain "sharpness" property of a sequence. If, for example, $|\rho_1|<|\rho_2|$, the M-TAR model exhibits little decay for positive Δy_{t-1} but substantial decay for negative Δy_{t-1}, i.e., increases tend to persist but decreases tend to revert quickly toward the attractor.

Asymmetric Unit-Root Test and Cointegration Test

Development of Asymmetric Unit-Root Test

Enders and Granger (1998) developed a new method which can be used to test the null hypothesis H_0: $\rho_1=\rho_2=0$ against the alternative of a TAR or an M-TAR model. They deduced the critical values via comprehensive Monte Carlo experiment. To realize this, they employ RATS software to generate 100,000 random walk processes of the following form for each sample size T, i.e.,

$$(4.10) \quad y_t = y_{t-1} + \varepsilon_t \quad t=1,\ldots\ldots T$$

where T= 50, 100, 250, and 1000.

For each sample size T, a total of T+100 normally distributed and uncorrelated pseudo-random numbers with standard deviation equal to unity were drawn to represent the $\{\varepsilon_t\}$ sequence. Setting the initial value of the sequence (i.e., y_0) equal to zero, the remaining values of $\{y_t\}$ were generated using (4.10). For each of the 100,000 series, the first 100 realizations were discarded (to get rid of the effect of initial value). The TAR model given by (4.7) and (4.8) was estimated and F-statistic for testing $\rho_1=\rho_2=0$ was obtained. Then the

95% quantile of these F-statistics was found and this is the critical value for H_0: $\rho_1=\rho_2=0$ at 5% significance level. Enders and Granger call this critical value Φ-statistic. It's tabulated in Table 4.3.

The same procedure was repeated for an M-TAR model using the indicator function given by (4.9). The corresponding F-test statistic, called Φ^*, are reported in the right-hand side of Table 4.3.

Two additional sets of critical values are reported in Table 4.4. Table 4.4 is to test the null hypothesis of a random walk process with a non-zero sample mean against the alternative of TAR and M-TAR.

Table 4.3: Enders-Granger Critical Values -- Φ and Φ^* statistic

	Φ - statistic			Φ^* - statistic		
#	90%	95%	99%	90%	95%	99%
50	3.30	4.12	6.09	2.98	3.81	5.79
100	3.18	3.95	5.69	2.83	3.60	5.38

Table 4.4: Enders-Granger Critical Values: Φ_μ and Φ_μ^* statistic

	Φ_μ -statistic			Φ_μ^* -statistic		
#	90%	95%	99%	90%	95%	99%
50	3.84	4.73	6.85	4.17	5.14	7.43
100	3.79	4.64	6.57	4.11	5.02	7.10

Development of Asymmetric Cointegration Test

Enders and Siklos (1998) deduced critical values for testing if there is a cointegration relationship among two variables and three variables in the asymmetric framework respectively. They conducted Monte-Carlo approach to do this. To see how it works, consider two-variable case. 30,000 sets of random walk processes of the following form were generated:

$$(4.11)\quad x_{1t} = x_{1,t-1} + \varepsilon_{1t} \qquad t=1,\ldots\ldots T$$

$$(4.12)\quad x_{2t} = x_{2,t-1} + \varepsilon_{2t} \qquad t=1,\ldots\ldots T$$

For T=100 and 500, two sets of T normally distributed and uncorrelated pseudo-random numbers with standard deviation equal to unity were drawn to represent the $\{\varepsilon_{1t}\}$ and $\{\varepsilon_{2t}\}$ sequences. Setting the initial values of x_{1t} and x_{2t} equal to zero, the next T values of each were generated using (4.11) and (4.12). For each of the 30,000 series, the TAR model given by (4.13) (4.14) (4.15) was estimated:

$$(4.13)\quad x_{1t} = \beta_2 x_{2t} + \mu_t$$

$$(4.14)\quad \Delta\mu_t = I_t \rho_1 \mu_{t-1} + (1-I_t)\, \rho_2 \mu_{t-1} + \varepsilon_t$$

where

$$(4.15)\qquad I_t = \begin{cases} 1 & if \quad \mu_{t-1} \geq 0 \\ 0 & if \quad \mu_{t-1} < 0 \end{cases}$$

and the F-Statistic for the null hypothesis $\rho_1 = \rho_2 = 0$ were tabulated in Table 4.5. These critical values can be used to test the null hypothesis of a unit-root process against the alternative of a TAR model.

Also note that the distribution of Φ-statistic depends on sample size, the number of lagged changes included in the dynamic adjustment equation, and the number of variables in the cointegrating relationship. See Table 4.5 for the different critical values corresponding to each combination of these factors.

The Monte-Carlo experiment was repeated for an M-TAR model. The corresponding test statistics- called $\Phi(M)$- are reported in Table 4.6.

Table 4.5: Enders-Siklos Critical Values-- Distribution of Φ

	Two Variable Case								
	No Lagged Change			One Lagged Change			Four Lagged Changes		
#	90%	95%	99%	90%	95%	99%	90%	95%	99%
100	5.04	6.07	8.20	4.99	5.98	8.21	4.94	5.91	8.22

Table 4.6: Enders-Siklos Critical Values--Distribution of $\Phi(M)$

	Two Variable Case								
	No Lagged Change			One Lagged Change			Four Lagged Changes		
#	90%	95%	99%	90%	95%	99%	90%	95%	99%
100	5.52	6.57	9.04	5.43	6.45	8.75	5.34	6.35	8.73

The Power Comparison Between Conventional Test Methodology and New Test Methodology

The major flaw of the conventional unit-root test and cointegration test is the low power. Hence, it is of great interest to compare the power of the new methodology to the power of more traditional Dickey-Fuller test and Engle-Granger test.

Dickey-Fuller Test and Enders-Granger Test

To compare the powers of these two tests, for various values of ρ_1 and ρ_2, 2500 series were generated using:

$$(4.16) \quad \Delta y_t = I_t \rho_1 [y_{t-1} - a_0] + (1 - I_t) \rho_2 [y_{t-1} - a_0] + \varepsilon_t$$

where

$$(4.17) \quad I_t = \begin{cases} 1 & if \quad y_{t-1} \geq a_0 \\ 0 & if \quad y_{t-1} < a_0 \end{cases}$$

for T=100.

Each series was regressed on a constant and save the residuals $\{\bar{y}_t\}$. For each of the resulting $\{\bar{y}_t\}$ series, the following regression equation was estimated:

$$(4.18) \quad \Delta \bar{y}_t = I_t \rho_1 \bar{y}_t + (1 - I_t) \rho_2 \bar{y}_t + \bar{\varepsilon}_t$$

For each of the 2500 regressions, the sample Φ_μ statistics were calculated and compared to the appropriate critical values. The percentage of times that the null hypothesis was correctly rejected is reported in the central portion of Table 4.7.

Table 4.7: Power Comparison between Dickey-Fuller Test and Enders-Granger Test

ρ_1	ρ_2	Φ_μ			DF		
		10%	5%	1%	10%	5%	1%
-0.05	-0.05	18.52	9.72	1.84	22.88	11.64	2.44
-0.10	-0.10	46.24	28.48	8.08	53.56	33.44	8.96
-0.10	-0.20	67.92	50.96	19.12	74.28	55.00	21.04
-0.10	-0.50	90.88	80.44	48.96	92.36	81.20	48.80
-0.10	-0.75	94.32	87.36	60.60	94.16	86.48	57.88
-0.10	-1.50	98.44	96.04	82.60	98.40	94.96	77.24

For comparison, we estimate the following equation for each of the 2500 generated sequences:

$$(4.19) \qquad \Delta y_t = a_0 + \rho y_{t-1} + \varepsilon_t$$

The t-statistic for the null of $\rho = 0$ was compared to Dickey-Fuller τ_μ statistic. The percentage of times the Dickey-Fuller test correctly rejected the null hypothesis is shown in the right portion of Table 4.7.

Inspection of Table 4.7 indicates that in general, Dickey-Fuller test dominates Enders-Granger test in terms of power. This is true even in the presence of significant asymmetry. The performance of Enders-Granger test is better with the existence of extreme asymmetry. e.g., when $\rho_1 = -0.10$, $\rho_2 = -1.50$, it performs slightly better than Dickey-Fuller test.

The poor performance of Enders-Granger test is due to the use of a 2-step procedure and to the estimation of one additional coefficient as compared to the Dickey-Fuller test. The resulting power loss dominates the gain from correctly specifying the model.

Table 4.8 reports results of power comparison using M-TAR model. Notice that the power of Enders-Granger test is often substantially larger than that of Dickey-Fuller test. However, when the true adjustment process is symmetric, the Dickey-Fuller test has great power than Enders-Granger test.

Table 4.8: Power Comparison between Dickey-Fuller Test and Enders-Granger Test

ρ_1	ρ_2	Φ_μ^*			DF		
		10%	5%	1%	10%	5%	1%
-0.025	-0.05	17.76	9.08	2.48	18.04	9.92	2.04
-0.025	-0.10	32.00	18.88	4.72	28.48	15.12	3.56
-0.025	-0.20	76.64	60.68	26.36	60.60	38.60	11.12
-0.05	-0.05	20.84	11.20	2.72	22.88	11.64	2.44
-0.05	-0.10	35.92	21.52	5.80	36.20	20.92	5.60
-0.05	-0.20	76.28	58.96	25.08	68.64	46.52	13.96
-0.10	-0.10	45.40	28.40	7.60	53.56	33.44	8.96
-0.10	-0.20	79.04	63.48	25.96	80.72	62.60	24.92
-0.10	-0.50	100.00	99.92	98.88	99.92	99.28	90.72

Engle-Granger Test and Enders-Siklos Test

Similar power comparison between Engle-Granger test and Enders-Siklos test was conducted and the results are reported in Tables 4.9 and 4.10. Note that the values in Table 4.9 and Table 4.10 are the number of instances that the null hypothesis was correctly rejected out of 500 times.

Table 4.9:Power Comparison between Engle-Granger test and Enders-Siklos test. (TAR)

ρ_1	ρ_2	Φ			Engle-Granger		
		10%	5%	1%	10%	5%	1%
-0.05	-0.05	8	4	2	11	3	0
-0.05	-0.10	17	7	2	24	8	1
-0.05	-0.25	66	27	3	84	28	2
-0.10	-0.10	59	22	3	90	30	3
-0.10	-0.25	310	151	22	351	191	19
-0.25	-0.25	500	497	387	500	499	382

Table 4.10: Power Comparison between Engle-Granger test and Enders-Siklos test. (M-TAR)

ρ_1	ρ_2	$\Phi(M)$			Engle-Granger		
		10%	5%	1%	10%	5%	1%
-0.05	-0.05	14	5	0	11	3	0
-0.05	-0.10	57	22	4	32	8	1
-0.05	-0.25	439	362	208	274	113	11
-0.10	-0.025	79	42	6	11	4	0
-0.10	-0.10	64	25	2	90	30	3
-0.10	-0.25	465	398	177	439	306	42
-0.25	-0.25	500	495	328	500	499	382

The overwhelming impression from Table 4.9 is that the power of the Engle-Granger test usually exceeds that of the Φ-statistic. But the situation changes a lot for the M-TAR model. From Table 4.10, we conclude:

(1) If adjustment is truly symmetric, the power of the Engle-Granger test generally exceeds that of the $\Phi(M)$ statistic in that the Enders-Siklos method entails the needless estimation of an additional coefficient with a consequent loss of power.

(2) If adjustment is truly asymmetric, Enders-Siklos test dominates Engle-Granger test because of the correct specification of the model. Moreover, with the increase of the degree of asymmetry, the relative power of Enders-Siklos method increases as well.

CHAPTER 5. TESTING FOR PPP IN ASYMMETRIC FRAMEWORK

Introduction

As indicated in chapter 2, even in the long run, empirical tests do not deliver strong support of PPP. The problem could come from two perspectives: first, conventional unit root tests (such as the Dickey-Fuller test) and cointegration tests (such as the Engle-Granger test) presuppose symmetric adjustment. As we have already shown in chapter 3, the assumption of symmetric adjustment seems inconsistent with behavior of national price levels. Secondly, national price indices (such as the CPI or GDP deflator) are each constructed by a national government using domestic weights. As such, changes in relative prices can cause measured deviations from PPP. Taking these two observations into account, we employ a new unit root test developed by Enders and Granger (1998) and a new cointegration test developed by Enders & Siklos (1998) as we introduced in chapter 4 to apply to a price level data that are constructed using the same weights across countries. One rationale behind PPP is the inter-country commodity arbitrage, which acts as an error correction mechanism to keep the exchange rate and/or prices within a small range around an equilibrium attractor. The Enders and Granger (1998) and Enders and Siklos (1998) tests are particularly designed to incorporate the asymmetry of adjustment process. The use of the appropriately weighted price level data can avoid some of the measurement problems involved in most other studies.

The remainder of this chapter is organized as follows. Section 2 develops the basic PPP and provides a review of conventional empirical testing methodology of PPP. Section 3 describes testing procedure allowing for asymmetry based on the observance of asymmetric price adjustment. Specifically, threshold autoregressive (TAR) and momentum threshold autoregressive (M-TAR) tests for unit-roots and cointegration are discussed. Section 4 provides the detailed data description. Preliminary test results and interpretation of the results are given in Section 5. Section 6 is about error-correction estimation and impulse response function analysis. Section 7 explains why PPP fails in Germany-US case and Germany-Canada case.

Basic Model

Consider the following econometric model of Purchasing Power Parity:

(5.1) $e_t = \alpha + \beta_1 p_t + \beta_2 p_t^* + \mu_t$

where

e_t = logarithm of the domestic currency price of foreign exchange in period t relative to a base year;

p_t = logarithm of the domestic price level in period t;

p_t^*= logarithm of the foreign price level in period t;

μ_t is a stochastic disturbance representing the deviation from PPP.

The strong version of PPP implies that $\alpha = 0$, $\beta_1 = -\beta_2 = 1$ and that μ is stationary. However, in practice, the homogeneity restriction is often relaxed due to the presence of transportation costs and other trade barriers. Moreover, $\beta_1 = -\beta_2 = 1$ is often relaxed also due to the possibility measurement error arising from such factors as using different weights in constructing the price indices. Equation (5.1), without imposing the coefficient restrictions, is called weak-PPP.

Existence of PPP is equivalent to the stationarity of μ_t. If μ_t is not stationary, deviations from PPP contain a permanent component, such that the discrepancy from PPP is never fully eliminated. Instead, if μ_t is stationary, any deviation from PPP cannot persist forever.

There are a number of different methodologies used to test for PPP. Among them the two most frequently used approaches are unit-root test and cointegration test. Unit-root test checks if the real exchange rate follows a random walk. If we reject the null hyporthesis of random walk, we conclude that PPP holds, otherwise PPP is rejected. The other most popular approach is to use cointegration test to determine whether there is a long-run equilibrium relationship between price levels and the exchange rate. It is widely acknowledged that e_t, p_t and p_t^* are I(1) variables. Therefore, if μ_t is I(0), a linear combination of the three variables can be found that is stationary. Hence, for PPP to hold, there must exist a cointegration relationship between prices and the exchange rate. It is worth noting here: first, these two approaches both explicitly take into account the possible nonstationarity of prices and exchange rate, they both check if there is a unit root in the error term to equation (5.1). Second, these two approaches differ in that the former methodology imposes $\alpha = 0$ and $\beta_1 = -\beta_2 = 1$ and the latter methodology estimates these parameters.

The standard unit-root test is Dickey-Fuller test. To perform, first, do the following OLS regression:

$$(5.2) \qquad \Delta r_t = \rho r_{t-1} + \varepsilon_t$$

where r_t is the period-t real exchange rate (usually in logarithms). The null hypothesis is $\rho = 0$, which implies that $\{r_t\}$ sequence follows random walk. The critical values can be found from Table 4.1.

If there is serial correlation, equation (5.2) can be modified as:

$$(5.3) \qquad \Delta r_t = \rho r_{t-1} + \Sigma \rho_{i-1} \Delta r_{t-i} + \epsilon_t$$

Diagnostic checks, conventional F-tests, and/or model selection criteria such as the *AIC* or *SBC* a can be used to determine the appropriate lag length.

The standard cointegration tests are Engle and Granger (1987) and Johansen (1995). Engle and Granger test follows a three-step procedure:

Step 1) Pretest the variables for their order of integration. (Dickey-Fuller test)

Step 2) Estimate the long-run equilibrium relationship as stated in equation (5.1)

Step 3) Perform the following OLS regression:

$$(5.4) \qquad \Delta \mu_t = \rho \mu_{t-1} + \epsilon_t$$

where the $\{\mu_t\}$ sequence is the regression residuals from equation (5.1). The parameter of interest in equation (5.4) is ρ, if we fail to reject the null hypothesis $\rho = 0$, we can conclude that the exchange rate and the prices are not cointegrated so that PPP fails. The critical values are provided in Table 4.2.

If the residuals of equation (5.4) do not appear to be white-noise, an augmented equation can be used instead of equation (5.4). Diagnostic checks and AIC/SBC criteria can be used to determine the lag length in the following autoregression:

$$(5.5) \qquad \Delta \mu_t = \rho \mu_{t-1} + \Sigma \rho_{i-1} \Delta \mu_{t-i} + \epsilon_t$$

Similarly, the Johansen (1995) methodology begins with a specification of the form:

$$(5.6) \qquad \Delta x_t = \pi x_{t-1} + v_t$$

where x_t is the (3 x 1) vector $(e_t\ p_t\ p_t^*)^T$.

π is a (3 x 3) matrix.

and v_t is a (3 x 1) vector of stationary disturbances that may be contemporaneously correlated.

The first step is to estimate π and to determine its rank. Equation (5.6) can take many different forms including the introduction of deterministic regressors, the addition of lagged changes in Δx_t. If we cannot reject the null of zero rank, we fail to reject the hypothesis that the prices and exchange rate are not cointegrated.

Asymmetric Unit-Root Test and Cointegration Test

Both Dickey-Fuller test and Engle-Granger test imply symmetric adjustment. Taking asymmetric price adjustment into consideration, the dynamic relationships implicit in the Dickey-Fuller (1979), Engle and Granger (1987) and Johansen (1995) are misspecified.

Enders and Granger (1998) and Enders and Siklos (1998) developed a number of unit-root and cointegration tests that do not presuppose a linear-symmetric adjustment mechanism. They use comprehensive Monte-Carlo simulation to deduce the critical values. One of the appealing properties of these asymmetric models is the enhanced power of the test, compared with the conventional test when the true scenario is asymmetric. After all, many people attribute the empirical failure of PPP to the low power of standard test in discriminating between non-stationary versus near stationary processes. i.e., the issue of whether real rates are slow-decaying or unit-root process is hard to tell using only the conventional test methodology. A very important motivation of employing the asymmetric models is that this enhanced power may swing the conclusion of whether the PPP holds empirically.

There are two variations for the asymmetric models depending on different Heaviside indicators. Namely, they are TAR (Threshold AutoRegressive) model and M-TAR(Momentum-Threshold Autoregressive) model. In the TAR model, the deviations from the long-run equilibrium in equation (5.4) behave as a Threshold Autoregressive process. Consider modifying equation (5.2) such that:

$$(5.7) \qquad \Delta r_t = I_t \, \rho_1 r_{t-1} + (1 - I_t)\rho_2 r_{t-1} + \epsilon_t$$

where: I_t is the Heaviside indicator:

$$(5.8) \qquad I_t = \left[\begin{array}{l} 1 \; if \, r_{t-1} \geq 0 \\ 0 \; if \, r_{t-1} < 0 \end{array} \right.$$

In the M-TAR model, Heaviside indicator is defined as follows:

$$(5.9) \qquad I_t = \left[\begin{array}{l} 1 \; if \, \Delta r_{t-1} \geq 0 \\ 0 \; if \, \Delta r_{t-1} < 0 \end{array} \right.$$

The threshold models allow for differential speeds of adjustment depending on the current state of the real exchange rate. Critical values for the test $\rho_1 = \rho_2 = 0$ are contained in Enders and Granger (1998). Similarly, the residuals from the Engle-Granger test can be tested using the two Threshold models. Consider estimating the residuals from equation (5.1) as:

$$(5.10) \qquad \Delta \mu_t = I_t \, \rho_1 \, \mu_{t-1} + (1 - I_t) \, \rho_2 \mu_{t-1} + \epsilon_t$$

where I_t is the Heaviside indicator such that:

$$(5.11) \qquad I_t = \begin{cases} 1 \ \text{if} \ \mu_{t-1} \geq 0 \\ 0 \ \text{if} \ \mu_{t-1} < 0 \end{cases}$$

or:

$$(5.12) \qquad I_t = \begin{cases} 1 \ \text{if} \ \Delta\mu_{t-1} \geq 0 \\ 0 \ \text{if} \ \Delta\mu_{t-1} < 0 \end{cases}$$

If we take a look at the TAR and M-TAR models, we know that the asymmetric adjustment is implied by different values of ρ_1 and ρ_2. For example, When r_{t-1} or Δr_{t-1} is positive, the adjustment is $\rho_1 r_{t-1}$, and if r_{t-1} or Δr_{t-1} is negative, the adjustment is $\rho_2 r_{t-1}$. It is sufficient to guarantee the convergence of $\{r_t\}$ if $-2 < (\rho_1, \rho_2) < 0$. The critical values for the null hypothesis $\rho_1 = \rho_2 = 0$ is tabulated by Enders and Granger (1998). Table 4.3 - 4.4. In addition, if the $\{r_t\}$ sequence is stationary, the least squares estimators of ρ_1 and ρ_2 have an asymptotic multivariate normal distribution (Tong 1983). Therefore, if the null hypothesis $\rho_1 = \rho_2 = 0$ is rejected, it is then possible to test for symmetric adjustment (i.e., $\rho_1 = \rho_2$) using a standard F-test. Similar statements can be made in cointegration test except for the fact that the cointegrating vector must be estimated from the data. The appropriate critical values are given by Enders and Siklos (1998). Table 4.5 and 4.6.

Finally, to avoid any serial correlation in the error-terms, we allow the lag terms of Δr_t and $\Delta\mu_t$ to enter into equation (5.7) and equation (5.10) respectively. Diagnostic checks of residuals and AIC/SBC criteria are used to determine the appropriate lag length for individual pair of countries.

Data Description

The data set we employ in the paper set is highly unusual in that it is particularly well suited to study PPP [the data was generously made available to us by John Pippenger]. The annual exchange rate and price level data are from *Internationaler Verleich der Preise fur die Lebenshaltung* published by the German Statistical Office (Statistisches Bundesamt). The series cover from 1927 to 1994 for a total of 68 observations. Because the data was published to adjust the salaries of German diplomats and foreign service people outside Germany, the price levels are constructed using weights that represent the typical spending pattern of a four-person family in Germany. Even though the price levels are constructed by only using cost of living in the capital of each country, this data has a very appealing property that makes it dominate other data sets in terms of testing for PPP, that is, equal weights are used for the goods in the basket on constructing the price index.

Why is it so essential to require equal-weight data? We know basically PPP comes from "law of one price" for each good. In the absence of trade barriers and transportation costs, inter-country commodity arbitrage ensures "law of one price", i.e., $ep_i^* = p_i$, where e is the domestic currency price of foreign currency; p_i^* and p_i are the prices for good i in the foreign country and domestic country, respectively. It is possible to show that if the weights that are used to construct the price indices are same in the two countries, the "law of one price" implies the validity of purchasing power parity.

Suppose there are two countries A and B. They both use the same basket of goods to construct the price indices. The goods in the basket are good i (i=1,2,.......m). Without losing any generality, we denote country A as domestic country and country B as foreign country. Then, we have:

$$(5.13)\quad \left[\begin{array}{l} P = P_{1A}^{\mu_{1A}} P_{2A}^{\mu_{2A}} \cdots\cdots P_{mA}^{\mu_{mA}} \\ P^{*} = P_{1B}^{\mu_{1B}} P_{2B}^{\mu_{2B}} \cdots\cdots P_{mB}^{\mu_{mB}} \end{array}\right.$$

where P_{ij} is the price level of good i in country j.

i=1,2,........m; j= A,B.

μ_{ij} is the weight put on good i in country j such that: $\Sigma\mu_{ij} = 1$.

By law of one price, we have: $eP_{iB} = P_{iA}$ i=1,2,......m.

It is possible to show that PPP holds for all possible prices (i.e., $eP^* = P$) if and only if $\mu_{iA} = \mu_{iB}$. The proof is straightforward. Substitute $p_{iA} = ep_{iB}$ and $\mu_{iA} = \mu_{iB}$ into the definition of P to obtain:

$$(5.14)\quad P = P_{1A}^{\mu_{1A}} P_{2A}^{\mu_{2A}} \cdots\cdots P_{mA}^{\mu_{mA}}$$

$$=(eP_{1B})^{\mu_{1B}}(eP_{2B})^{\mu_{2B}} \cdots\cdots (eP_{mB})^{\mu_{mB}}$$

$$= eP^*$$

since the sum of the weights equals unity.

Note that the typical tests of PPP do not use price levels constructed using equal weights. However, our data set uses equal weights for 221goods and services. We have data available on six countries (Canada, France, Germany, Italy, UK and US), hence have 15 different country pairs. For each pair of the countries, the level of exchange rate and the ratio of the level of the price index are published. Unfortunately, we do not have individual price levels for the various countries, instead, we have price ratios.

Test Procedure and Interpretation of the Results

The asymmetric TAR and M-TAR cointegration test is carried out using the following procedure:

Step 1) Apply OLS to estimate following regression:

$$(5.15) \qquad e_t = \alpha + \beta p_t + \mu_t$$

where: e_t= logarithm of the domestic currency price of foreign exchange in period t relative to a base year;

p_t = logarithm of the ratio of domestic price level and the foreign price level in period t;

μ_t is a stochastic disturbance representing the deviation from PPP.

Step 2) Perform the OLS regression on equation (5.10) and (5.11) or (5.12), where $\{\mu_t\}$ are the regression residuals from equation (5.15). Use diagnostic checks and AIC/SBC rule to determine the appropriate lag length of $\Delta\mu_t$. It is worth to note that threshold zero is appropriate because we argued in chapter 3 that price index displays positive asymmetry on the two sides of zero.

Step 3) Use Enders-Siklos critical values listed in Table 4.5 and 4.6 to test the null hypothesis of $\rho_1 = \rho_2 = 0$. Notice that the critical values depend on the lag length as well as the number of variables in the regression.

If the null of $\rho_1 = \rho_2 = 0$ is rejected, we move on to test the symmetric assumption $\rho_1 = \rho_2$ by using standard F-test.

Similarly, we can constrain the cointegrating vector such that we can test the stationarity of the real exchange rate in log form.

Interpretation of the Results

Tables 5.1 and 5.2 report the test results for symmetric Engle-Granger cointegration test and asymmetric (TAR and M-TAR) Enders-Siklos cointegration test of purchasing power parity, respectively. Tables 5.4, 5.5 and 5.6 are the unit root test results regarding the stationarity of the log real exchange rates. To be specific, Table 5.4 contains Dickey-Fuller test results, Table 5.5 reports Enders-Granger TAR model results while Table 5.6 reports Enders-Granger M-TAR model results.

Interpretation of Tables 5.1 and 5.2

(1) By looking at Table 5.1, we found that using Enger-Granger test, PPP holds for 10 out of 15 pairs of countries at 10% significance level, 7 at 5% level and 3 at 1% level. This is more positive evidenve in favor of PPP than conventional study. This improvement can be attributed to the better data set.

(2) If we compare Table 5.1 with 5.2, we can see that the results of symmetric approach and asymmetric approach are pretty much similar in that for any pair of countries, the three methods (Engle-Granger, Enders-Siklos TAR and Enders-Siklos M-TAR) almost always came up with the same conclusion with only one exception of France-Italy case. In France-

Table 5.1: Engle-Granger Cointegration Test of Purchasing Power Parity[a]

*Log form	Engle-Granger Test			
	ρ	E-G statistic[b]	lag[c]	AIC/SBC
CUS	-0.24	-3.09*	lag=1	-158.42 / -154.04
CF	-0.31	-3.73**	lag=2	11.65 / 18.18
CI	-0.32	-3.79**	lag=2	41.62 / 48.15
CUK	-0.15	-2.48	lag=1	-60.66 / -56.29
USF	-0.34	-3.98**	lag=2	4.62 / 11.14
USI	-0.39	-4.07***	lag=2	35.26 / 41.78
USUK	-0.22	-3.26*	lag=1	-62.77 / -58.40
DMUS	-0.11	-2.47	lag=1	-48.61 / -44.23
DMC	-0.1	-2.33	lag=1	-42.16 / -37.78
DMF	-0.42	-4.45***	lag=3	-18.17 / -9.53
DMI	-0.32	-3.38**	lag=2	30.07 / 36.60
DMUK	-0.16	-2.92	lag=1	-93.25 / -88.87
FI	-0.2	-2.69	lag=0	-2.83 / -0.63
FUK	-0.5	-4.64***	lag=3	-11.44 / -2.80
IUK	-0.3	-3.23*	lag=2	32.79 / 39.32

[a] * indicates 10% significance level. ** indicates 5% significancelevel. *** indicates 1% significance level.

[b] E-G statistic is the t-statistic for testing H_0: $\rho = 0$.

[c] Optimal lag length is obtained by searching through no lag to four lags via AIC/SBC minimization rule.

Table 5.2: Enders-Siklos TAR-Model and M-TAR Model Cointegration Test of Purchasing Power Parity

*log	TAR model			M-TAR model		
	ρ_1/ρ_2 p-value of $\rho_1=\rho_2$	Φ-statistic (lag)	AIC/ SBC	ρ_1/ρ_2 p-value of $\rho_1=\rho_2$	Φ(M) (lag)	AIC/ SBC
CUS	-0.32/-0.16 (0.28)	5.41* (lag=1)	-160.14/ -153.52	**-0.37/-0.09** **(0.05)**	**7.01**** **lag=1**	**-160.45** **/-153.9**
CF	**-0.37/-0.22** **(0.35)**	**7.38**** **(lag=2)**	**12.87/** **21.69**	-0.34/-0.24 (0.54)	7.08** lag=2	13.25/ 21.95
CI	-0.35/-0.25 (0.55)	7.30** (lag=2)	44.33/ 53.15	-0.55/-0.08 (0.00)	17.34*** lag=2	27.93/ 36.63
CUK	**-0.14/-0.15** **(0.89)**	**3.03** **(lag=1)**	**-59.67/** **-53.05**	-0.11/-0.19 (0.51)	3.26 lag=1	-59.13/ -52.56
USF	-0.40/-0.26 (0.39)	8.27*** (lag=2)	5.76/ 14.57	**-0.41/-0.22** **(0.26)**	**8.61**** **lag=2**	**5.27/** **13.97**
USI	-0.44/-0.26 (0.37)	8.65*** (lag=2)	37.26/ 46.08	**-0.56/-0.02** **(0.00)**	**14.54***** **lag=2**	**27.28/** **35.98**
USUK	-0.22/-0.22 (0.99)	5.24* (lag=1)	-61.79/ -55.17	**-0.18/-0.32** **(0.32)**	**5.84*** **lag=1**	**-61.84/** **-55.27**
DMUS	-0.12/-0.11 (0.94)	3.01 (lag=1)	-47.41/ -40.80	**-0.17/-0.07** **(0.28)**	**3.64** **lag=1**	**-47.81/** **-41.24**
DMC	**-0.11/-0.10** **(0.90)**	**2.67** **(lag=1)**	**-40.88/** **-34.26**	-0.14/-0.07 (0.45)	2.97 lag=1	-40.75/ -34.18
DMF	-0.61/-0.30 (0.06)	12.26*** (lag=3)	-21.56/ -10.54	**-0.61/-0.22** **(0.01)**	**14.20***** **lag=3**	**-23.06/** **-12.27**
DMI	-0.47/-0.15 (0.06)	7.73** (lag=2)	29.04/ 37.86	**-0.53/-0.09** **(0.00)**	**14.28***** **lag=2**	**18.14/** **26.83**
DMUK	**-0.16/-0.16** **(0.99)**	**4.19** **(lag=1)**	**-92.72/** **-86.11**	-0.15/-0.17 (0.87)	4.20 lag=1	-91.27/ -84.71
FI	-0.24/-0.13 (0.45)	3.88 (lag=0)	-1.43/ 2.98	**-0.35/-0.07** **(0.01)**	**8.18**** **lag=0**	**-8.91/** **-4.50**
FUK	**-0.42/-0.56** **(0.43)**	**11.02***** **(lag=3)**	**-11.07/** **-0.04**	-0.43/-0.55 (0.52)	10.87*** lag=3	-9.88/ 0.91
IUK	-0.16/-0.39 (0.18)	6.21** (lag=2)	33.62/ 42.44	**-0.10/-0.54** **(0.00)**	**14.16***** **lag=2**	**20.12/** **28.81**

Italy case, only M-TAR model found 5% significance, which means PPP can only be detected by using M-TAR model.

(3) However, we still can see asymmetric test methodology supports PPP more than Engle-Granger test. As a matter of fact, both of TAR model and M-TAR model provide at least as much evidence supporting PPP as Engle-Granger test. And moreover, stronger support can be found either from finding more cases for which PPP holds at the same significance level or from the ability to detect PPP in higher significance level for the same country pair.

(4) We choose the best fitting asymmetric model for each pair of country via AIC/SBC rule. They are marked by bold font in Table 5.2. Then we can make the following comparison as shown in Table 5.3, where the entries represent the number of pairs of countries for which PPP holds.

Table 5.3: Comparison Between Engle-Granger Test and Best-fitting Asymmetric Cointegration Test

	10% significance	5% significance	1% significance
Engle-Granger test	10	-	3
Best Fitting test (TAR or M-TAR)	11	10	6

(5) The optimal lag length are very consistent across these three tests.

(6) From AIC/SBC, we can see that Engle-Granger test has smaller AIC/SBC for those country pairs accepting null hypothesis $\rho_1 = \rho_2$. Asymmetric test has smaller AIC/SBC for those country pairs rejecting null hypothesis $\rho_1 = \rho_2$.

(7) At 10% significance level, PPP fails for 4 pairs of countries. They are Canada-UK, Germany-US, Germany-Canada, Germany-UK. It is interesting to note that first,Germany is involved in three cases. Secondly, three cases are between a North American country and an EC nation.

(8) If we compare the estimated values of ρ and ρ_1, ρ_2, we find that ρ is always in between ρ_1 and ρ_2.

Interpretation of Tables 5.4, 5.5 and 5.6

Table 5.4 is about the Dickey-Fuller test of the unit root in log real exchange rates. Table 5.5 is the corresponding TAR model Enders-Granger test while Table 5.6 is M-TAR model Enders-Granger test. A few comments can be made based on test results.

(1) With log transformation of the real exchange rates data, Dickey-Fuller test found significant evidence in favor of PPP. At 5% significance level, PPP holds in 9 out of 15 pairs of countries.

(2) For each of the 15 pairs, we choose the best fitting model from the two asymmetric alternatives via AIC/SBC rule. They are marked by bold font in Table 5.5 and 5.6. Then we can make the following comparison as shown in Table 5.7, where the entries represent the number of pairs of countries for which PPP holds.

It is straightforward to observe that asymmetric model works much better than Dickey-Fuller test at 1% significance level, though this dominance is weak at 10% and 5% level. And moreover, if we take a look at the AIC/SBC values, we found smaller AIC/SBC in best fitting model than in Dickey-Fuller model in all pairs of countries except for one case: France-UK. It

Table 5.4: Dickey-Fuller Unit-Root Test of the Log Real Exchange Rates[a]

*Log form	Dickey -Fuller			
	ρ	t-statistic[b]	lag[c]	AIC/SBC
CUS	-0.236	-3.09**	lag=1	-155.49 / -148.92
CF	-0.268	-3.43**	lag=2	14.79 / 23.48
CI	-0.329	-3.81***	lag=2	43.79 / 52.48
CUK	-0.071	-1.48	lag=1	-45.18 / -38.61
USF	-0.282	-3.53**	lag=2	8.65 / 17.35
USI	-0.391	-4.09***	lag=2	37.33 / 46.02
USUK	-0.093	-1.89	lag=1	-50.31 / -43.75
DMUS	-0.097	-2.28	lag=1	-51.57 / -45.00
DMC	-0.084	-1.99	lag=1	-44.38 / -37.81
DMF	-0.257	-3.34**	lag=3	-7.85 / 2.94
DMI	-0.317	-3.39**	lag=2	33.35 / 42.05
DMUK	-0.12	-2.41	lag=1	-85.39 / -78.82
FI	-0.187	-2.43	lag=1	0.92 / 7.48
FUK	-0.464	-4.48***	lag=3	-8.49 / 2.31
IUK	-0.299	-3.20**	lag=2	34.75 / 43.45

[a] * indicates 10% significance level, ** indicates 5% significancelevel, *** indicates 1% significance level.
[b] Dickey-Fuller statistic is the t-statistic for testing H_0: $\rho = 0$.
[c] Optimal lag length is obtained by searching through no lag to four lags via AIC/SBC minimization rule.

Table 5.5: Enders-Granger TAR-Model Unit Root Test of the Log Real Exchange Rates

*log

	ρ_1 / ρ_2 p-value of $\rho_1 = \rho_2$	Φ-statistic (lag)	AIC/ SBC
CUS	-0.273/-0.164 (0.45)	5.23** (lag=1)	-156.0 / -149.5
CF	**-1.257/-0.010 (0.00)**	**15.23*** (lag=2)**	**0.10 / 8.86**
CI	-0.668/-0.096 (0.00)	11.50*** (lag=2)	37.46/ 46.21
CUK	**-0.736/-0.031 (0.03)**	**3.31 (lag=1)**	**-49.13/ -42.56**
USF	**-1.594/-0.106 (0.00)**	**15.95*** (lag=2)**	**-6.64/ 2.12**
USI	**-0.877/-0.096 (0.00)**	**15.71*** (lag=2)**	**26.42/ 35.18**
USUK	**-0.745/-0.031 (0.02)**	**3.67 (lag=1)**	**-53.80/ -47.23**
DMUS	**-0.055/-0.406 (0.06)**	**3.84 (lag=1)**	**-53.75/ -47.18**
DMC	**-0.051/-0.375 (0.07)**	**3.54 (lag=1)**	**-46.99/ -40.43**
DMF	**-0.958/-0.183 (0.00)**	**15.27*** (lag=3)**	**-24.52/ -13.57**
DMI	-0.578/-0.141 (0.01)	9.67*** (lag=2)	26.98/ 35.74
DMUK	**-0.502/-0.059 (0.05)**	**3.66 (lag=1)**	**-86.79/ -80.23**
FI	-0.211/-0.142 (0.66)	3.09 (lag=1)	0.71/ 7.28
FUK	-0.407/-0.339 (0.75)	6.81** (lag=3)	-3.40/ 7.54
IUK	-0.186/-0.266 (0.72)	4.26* (lag=2)	36.78/ 45.54

Table 5.6 : Enders-Granger M-TAR Model Unit Root Test of the Log Real Exchange Rates

*log

	ρ_1 / ρ_2 p-value of $\rho_1 = \rho_2$	Φ^*-statistic (lag)	AIC/SBC
CUS	**-0.319/-0.110** **(0.15)**	**6.14**** **(lag=1)**	**-157.65/** **-151.08**
CF	-0.216/-0.019 (0.07)	4.97* (lag=2)	16.73/ 25.43
CI	**-0.370/0.049** **(0.00)**	**12.21***** **(lag=2)**	**35.93/** **44.62**
CUK	-0.034/-0.037 (0.96)	0.88 (lag=1)	-44.35/ -37.78
USF	-0.211/0.031 (0.02)	5.39** (lag=1)	10.68/ 17.25
USI	-0.328/0.062 (0.00)	8.16*** (lag=3)	35.76/ 46.56
USUK	-0.040/-0.028 (0.85)	0.89 (lag=1)	-48.35/ -41.79
DMUS	-0.087/-0.048 (0.57)	2.13 (lag=1)	-50.48/ -43.91
DMC	-0.068/-0.055 (0.85)	1.82 (lag=1)	-43.68/ -37.11
DMF	-0.449/0.053 (0.00)	14.23*** (lag=3)	-21.97/ -11.17
DMI	**-0.520/0.100** **(0.00)**	**15.06***** **(lag=2)**	**18.50/** **27.20**
DMUK	-0.134/-0.008 (0.15)	2.65 (lag=1)	-84.88/ -78.31
FI	**-0.333/0.094** **(0.01)**	**7.48***** **(lag=1)**	**-7.17/** **-0.60**
FUK	**-0.211/-0.349** **(0.46)**	**5.86**** **(lag=2)**	**-2.87/** **5.83**
IUK	**0.101/-0.476** **(0.00)**	**10.69***** **(lag=2)**	**25.31/** **34.01**

is worth noting that for this case, the p-value of testing H_0: $\rho_1 = \rho_2$ is 0.75 for TAR model and 0.46 for M-TAR model.

Table 5.7: Comparison between Dickey-Fuller test and best-fitting asymmetric unit-root tset

	10% significance	5% significance	1% significance
Dickey-Fuller test	9	9	3
Best Fitting test (TAR or M-TAR)	10	10	8

(3) For all 10 cases we reject H_0: $\rho_1 = \rho_2 = 0$ at 5% level, we move on to test symmetric hypothesis $\rho_1 = \rho_2$. We reject the null of symmetry for 8 cases at 1% level. The results show that asymmetry is common.

Asymmetric Error-Correction and Impulse Responses

The positive finding of cointegration with asymmetric adjustment justifies the estimation of the asymmetric error correction model shown in the Table 5.8.

The key feature in the Table 5.8 is the pattern of the estimated coefficients for z_plus and z_minus. In all three cases, the t-statistics imply that the responsiveness of the exchange rate to error correction terms (both of positive and negative) are significant. e.g.,Canada$/Lira rate falls(rises) whenever it lies above(below) its long-run PPP level. The point estimates indicate that this rate adjusts by 96.2% of a positive gap from long-run PPP and 55.4% of a negative gap.

However, the interpretation of the price-ratio equation is somewhat difficult. The price ratio increases by 11.3% of a one-unit positive gap and increases by 43.3% of a one-unit negative gap from long-run PPP. Thus, the price ratio adjusts in the "wrong" direction in response to a negative discrepancy from long-run PPP. Overall, positive discrepancies are eliminated in a fashion that is substantially different from negative discrepancies. In response to a positive gap, the Canada$/Lira rate declines and the Canada/Italy price ratio rises. Both adjustments act to eliminate the discrepancy from PPP. On the other hand, in response to a negative gap, the Canada$/Lira rates moves more slighly and the price ratio even moves in the "wrong" direction. As such, there is less tendency for the negative gap to close.

Table 5.8: Estimates of the Error-Correction Models[a]

Canada-Italy Case

$$\underset{}{\Delta e_t} = \underset{(-2.13)}{-0.056} + \underset{(0.70)}{0.097\Delta e_{t-1}} - \underset{(-0.50)}{0.108\Delta p_{t-1}} - \underset{(-5.27)}{0.962z\_plus_{t-1}} - \underset{(-4.11)}{0.554z\_minus_{t-1}}$$

$$\Delta p_t = \underset{(-2.71)}{-0.058} - \underset{(-0.08)}{0.009\Delta e_{t-1}} + \underset{(2.00)}{0.349\Delta p_{t-1}} + \underset{(0.76)}{0.113z\_plus_{t-1}} - \underset{(-3.94)}{0.433z\_minus_{t-1}}$$

US-Italy Case

$$\Delta e_t = \underset{(-2.12)}{-0.057} + \underset{(0.73)}{0.110\Delta e_{t-1}} - \underset{(-0.36)}{0.080\Delta p_{t-1}} - \underset{(-4.72)}{0.997z\_plus_{t-1}} - \underset{(-3.77)}{0.560z\_minus_{t-1}}$$

$$\Delta p_t = \underset{(-2.46)}{-0.053} + \underset{(0.01)}{0.002\Delta e_{t-1}} + \underset{(2.26)}{0.406\Delta p_{t-1}} + \underset{(1.06)}{0.182z\_plus_{t-1}} - \underset{(-3.39)}{0.407z\_minus_{t-1}}$$

Italy-UK Case

$$\Delta e_t = \underset{(1.19)}{0.031} - \underset{(-0.13)}{0.021\Delta e_{t-1}} + \underset{(0.92)}{0.199\Delta p_{t-1}} - \underset{(-2.62)}{0.386z\_plus_{t-1}} - \underset{(-4.09)}{0.876z\_minus_{t-1}}$$

$$\Delta p_t = \underset{(1.50)}{0.033} - \underset{(-0.25)}{0.034\Delta e_{t-1}} + \underset{(2.91)}{0.538\Delta p_{t-1}} - \underset{(-2.32)}{0.293z\_plus_{t-1}} + \underset{(1.12)}{0.205z\_minus_{t-1}}$$

[a] In this Error-Correction estimation, we use unbiased threshold via Chang's method.

The interpretation of the US-Italy case is pretty much similar to the Canada-Italy situation. However, Italy-UK case is kind of different in the sense that the Italy/UK price ratio adjusts in the "wrong" direction in response to a positive gap rather than a negative gap. Hence, for Italy-UK circumstance, the negative gap can be more easily to be closed than positive gap.

In order to gain additional insight into the price ratio and exchange rate dynamics implied by equations in Table 5.8, let's take Italy/UK as example and consider the impulse response functions shown in Figure 5.1 and Figure 5.2. As shown in Figure 5.1, a positive one-standard-deviation Lira/pound rate shock opens a positive gap from long-run PPP, by observing the coefficients in error-correction equation, this gap leads to the depreciation of Lira/pound rate and decrease of the Italy/UK price ratio. The depreciation tendency of the rates narrows gap while the down-ward movement of the price ratio makes the gap even wider. However, by looking at the Figure, we conclude that the rate depreciates much faster than the drop of price ratio. i.e., the first tendency overpowers the second tendency, which means the gap is eliminated gradually overall.

Figure 5.2 depicts the dynamic behavior of the rate and price ratio in response to one-standard-deviation shock of the Italy/UK price ratio. Obviously, this shock will open a negative gap from long-run PPP. The error-correction equation in Table 5.8 tells us that in response to this negative gap, the Lira/pound rate will go up and the Italy/UK price ratio will go down. This is consistent with the curvature in Figure 5.2 if you restrict yout attention from period zero to period one. At period one, there is a turning point for the exchange rate. What could happens here is that at this point, the negative gap suddenly becomes positive gap due to the appreciation of the exchange rate as well as the drop of the price ratio. Because both of these tendencies work

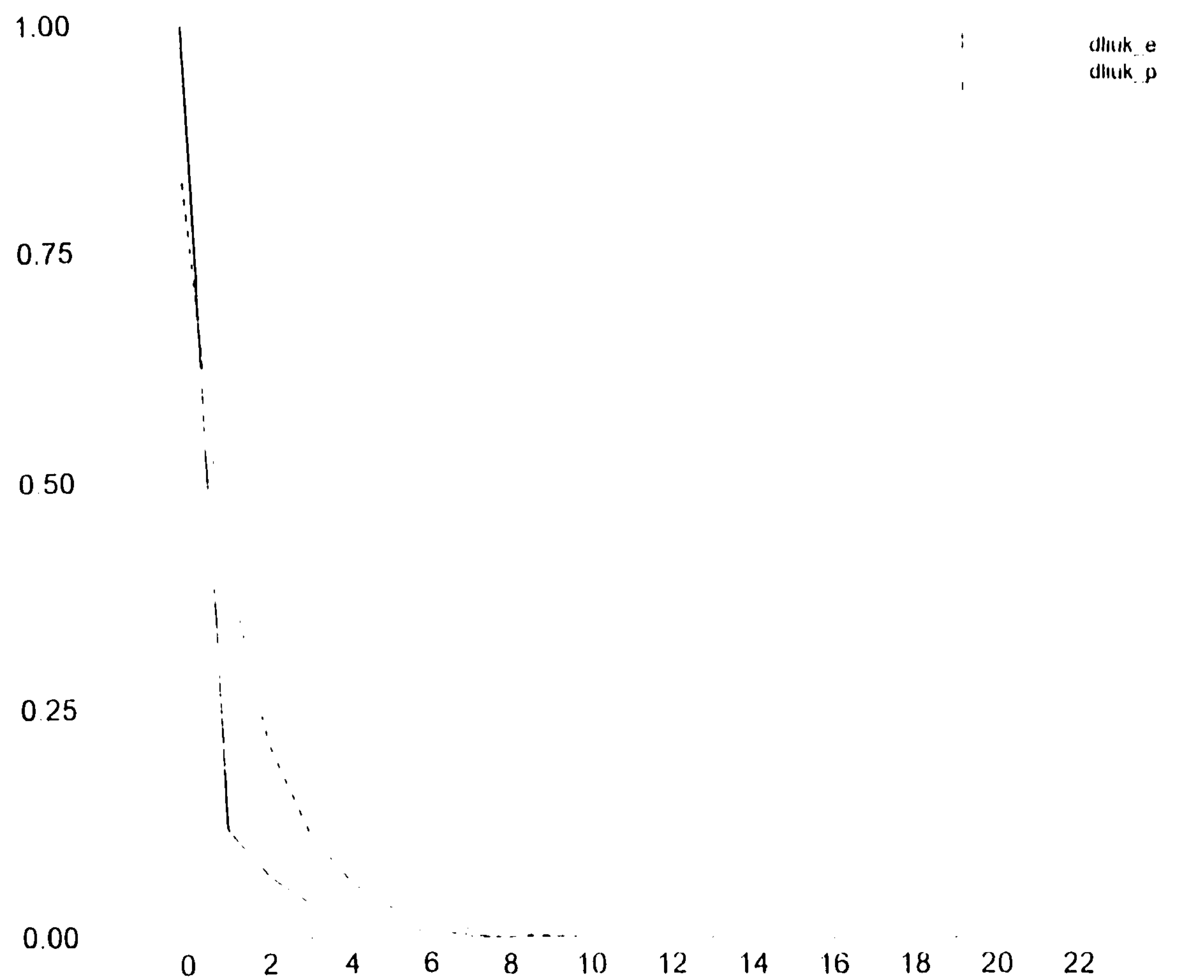

Figure 5.1: Plot of Responses to dliuk_e

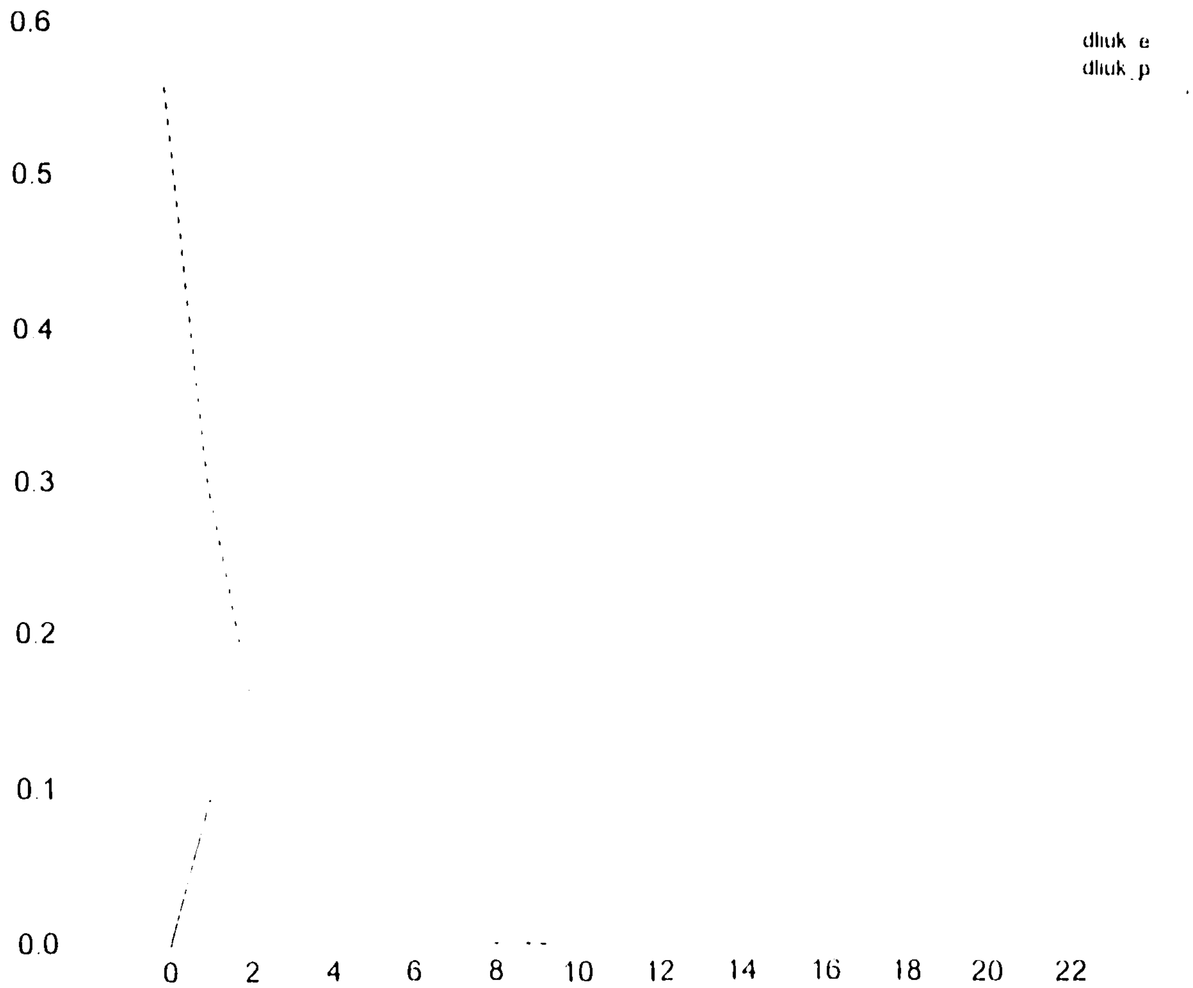

Figure 5.2: Plot of Responses to dliuk_p

in the same direction, it's very likely that the correction maybe overshooted such that a positive gap replaces the negative one. Once the gap becomes positive, the exchange rate goes down while the down-ward movement of the price ratio remains unchanged. But if we take a look at the adjustment speed of these two offsetting tendencies, we know that the exchange rate movement dominates the price-ratio movement, which closes the positive gap gradually. That's exactly what we can observe from the Figure.

Why PPP Fails in Germany-US Case and Germany-Canada Case?

General Explanation

Our data set consisits of six countries, which enable us to do PPP test for 15 pairs of nations. Out of these six countries, four of them are EC nations, the two others are North American nations. We found PPP holds between US and Canada as well as between any two European countries except for Germany-UK. According to the cointegration test results, out of the four failure cases at 10% significance level, Germany is involved in three. Another interesting thing is three of these four cases (except for Germany-UK) happened between an EC nation and a north American country.

A careful consideration about this will let us know it is not a coincidence. First of all, if there is any pair for which PPP tends to fail, it is most likely to happen between a north American country and an European country. Why? after all, the most straightforward reason for PPP failure is the deviation from law of one price. Geography distance as well as the border between two nations incur transportation cost and various trade barriers, which is a straight driving force for deviation from PPP. Engel and Rogers (1994) find that the variability in the price of a good in two different locations within a country depends on the distance between locations. Moreover, they find that holding other variables (including distance) constant, the variablity in prices between two U.S. or two Canadian cities is much less than between a Canadian and a U.S. city. Crossing the U.S. -Canadian border adds as much to the variability of prices as adding (a minimum) of 2500 miles between cities within a country.

So we know distance is important, and crossing the border might be even more. Since the border between U.S. and Canada is equivalent to a 2500 miles, I would guess the border between Germany and Canada or U.S. worths much more. In addition, if you consider the whole Atlantic Ocean seperating Europe and North America, we should not feel too strange if we see PPP fails between a north America nation and an European nation.

But why Germany is involved in most of failure cases? It seems that in addition to the distance and border factor, (particularly when you think about Germany-Uk), there got to be some other reasons. We propose here the high independence of the central bank of Germany might be a key factor. We know the first priority of the Germany central bank is to control the inflation and keep the stability of the macroeconomy. As Dornbusch (1982) showed, there is a trade-off between stability of the real exchange rate and price stability. If price stability is more prefered, then the stability of the real exchange rate could be sacrificed.

Consider a case in which the German domestic price goes up, to fight against inflation, the central bank will tighten economy by raising interest rate, which in turn trigger a capital inflow and lead to a positive balance of payments. Hence the Mark should appreciate, which deviates PPP furtherly. This simple mechanism supplys a brief explanation about how the central bank's priority goal could lead to the long run deviation from PPP.

A Model Approach about the Trade-Off between the Stability of Real Exchange Rate and the Stability of Prices

By extending Taylor (1979) model, Dornbusch (1982) shows in the following that exchange rate rules that closely follow purchasing power parity do offset the employment effects of price disturbance, but they do so at the cost of increased instability of prices.

Basic Model

We assume an economy where employment is determined by demand and where the unit labor requirement is a constant α. Denoting employment by N, we have

$$(5.16) \qquad N = \alpha(D + M^*)$$

where D and M* are domestic and foreign demand for home output. Domestic price is determined by unit labor costs:

$$(5.17) \qquad P = \alpha W$$

The trade balance is the difference between export receipts and import spending:

$$(5.18) \qquad TB = PM^* - P^*eM$$

where P* is the given foreign currency price of our imports, assumed constant and equal to unity, and e is the exchange rate. In addition, we assume:

$$(5.19) \qquad D = D(P/e,\ WN/e,\ H/e); \quad M = M(P/e,\ WN/e,\ H/e); \quad M^* = M^*(P/e)$$

where H is the money balance. So we assume the foreign import demand M* depends on the relative price (P/e), while both of domestic demand D and domestic import demand M depend on realtive price P/e, income WN/e and money balance H/e.

Combine (5.16), (5.17) and (5.19), we have:

$$(5.20) \qquad N = N(W/e, H/e) \qquad N_1 < 0, N_2 > 0.$$

The interpretation is: given the exchange rate and wage rates, a rise in the nominal money stock raises real balances, spending and employment. On the other hand, given money and the exchange rate, a rise in wages raises the relative and absolute price of domestic goods. With substitution effects dominating the real income effect, employment declines. Also we assume domestic exchange depreciation raises employment.

Likewisely, we can write:

$$(5.21) \qquad TB = TB(W/e, H/e) \quad T_1 < 0, T_2 < 0.$$

Accommodation and Macroeconomic Stability

This part of work extends Taylor's model to the open economy, it can be shown that the monetary and exchange rate policies that accommodate price disturbances will tend to stabilize output, but they do so at the cost of increased persistence in wage and price disturbances. The intuition is that the wage-setting process is influenced by expectations about the extent to which policies are accommodating. The more policies are expected to accomodate, the less labor has to be concerned about the unemployment consequences of wage policy and hence the slower is the adjustment of wages and prices.

The model studies deviations of output, relative prices and real balances from their long-run equilibrium levels. Let y, h, p and e denote the logs of real output, nominal money, domestic prices and exchange rate respectively. Without losing any generality, we assume in long-run equilibrium $y = p = e = h = 0$. We claim a key equation as follows;

$$(5.22) \quad y = a(e - p) + b(h - p) + v \qquad a, b > 0$$

which says that equilibrium output is determined by the real exchange rate e-p, real balances h-p, and a random term v. Note that (5.22) is simply a linear version of (5.20).

Accomodation policy rules are shown as:

$$(5.23) \quad h = \beta p \qquad e = \gamma p$$

where β and γ represent the elasticities of the nominal money stock and the nominal exchange rate with respect to the price level. Thus if the price level doubled the authorities would accomodate the price disturbance by a $100\beta\%$ increase in nominal money and a $100\gamma\%$ increase in the exchange rate.

Substitute (5.23) into (5.22), we have:

$$(5.24) \quad y = -\theta p + v \qquad \theta \equiv a(1-\gamma) + b(1-\beta)$$

(5.24), the reduced form equation for output, shows that an increase in the level of prices will reduce demand and output unless money and the exchange rates are fully accomodating, i.e., $\beta = \gamma = 1$. With partial accomodation (β, $\gamma<1$), a rise in prices reduces real balances and appreciates the real exchange rate, therefore lowering demand for domestic output. The cofficient θ represents the responsiveness of output to price disturbances and varies between zero and one depending on the extent of accomodation.

Taylor's model uses long-term, overlapping wage contracts combined with rational expectations as the framework for wage-price setting. In this model, there are two-period contracts and two groups, each having a contract coming up every other year for renegotiation. In this manner there are always two contracts overlapping, one in its first year, the other in its second year. The group that sets a new wage has three points to notice. One is the ongoing second year contract that is still in force. The second is the new contract. The third point to consider is the unemployment consequence of current wage setting.

The current wage contract x will thus be set with reference to ongoing contracts entered into last period, x_{-1}, expected future contracts, x^e_{-1}, and expected employment y^e and y^e_{-1} during the length of the current contract. Here the superscript e denotes the expectation operator and u is an error term:

$$(5.25) \quad x = (1-d)x_{-1} + d\,x^e_{-1} + \delta[(1-d)y^e + d\,y^e_{-1}] + u$$

where δ represents the responsiveness of the wage contract to employment prospects. Note from (5.25) that wages are purely forward looking if d =1 and purely backward looking for d = 0. The term u represents random movements in wages.

The price level is proportional to the average wage. The average wage in turn is formed by the currently effective contracts, x and x_{-1}. Thus the price level can be set as:

$$(5.26) \quad p = 0.5(x + x_{-1})$$

Now let us incorporate rational expectation into our model. Awaring of the fact that the rational expectation is a model-consistent expectation, we can derive from (5.24) and (5.26) by taking expectations:

$$(5.27) \quad y^e = -0.5\theta(x + x_{-1}) \qquad y^e_{-1} = -0.5\theta(x + x^e_{-1})$$

Combine (5.27) and (5.25), we get:

$$(5.28) \quad x^e = (1-d)x^e_{-1} + dx^e_{-1} - \delta[0.5(1-d)\theta(x^e + x^e_{-1}) + 0.5d\theta(x^e + x^e_{-1})]$$

Collecting terms gives us an equation for wages:

$$(5.29) \quad (1-d)x^e_{-1} - c\,x^e + d\,x^e_{-1} = 0, \text{ where } c \equiv (1+0.5\theta\delta)/(1-0.5\theta\delta)$$

with a solution:

$$(5.30) \quad x = \rho x_{-1} + u. \quad \rho \equiv (1/2d)\ \{c-[c^2-4d(1-d)]^{1/2}\}$$

Substitution of (5.30) into (5.26) yields:

$$(5.31) \quad p = \rho p_{-1} + 0.5(u+u_{-1})$$

Note that we can conclude $1 \leq c$ based on $0 \leq \theta < 1$. Furtherly we can claim that ρ achieves its maximum value when c=1 due to the fact that $\rho_c < 0$. That is to say, If the accommodation is fully, i.e., if c=1, then the price disturbances are the most persistent.

In summary, a policy of maintaining near purchasing power parity implies that we follow price disturbances by accommodating exchange rate adjustments. Such a policy keeps the real exchange rate relatively constant and thus reduces the source of employment variation. But because it prevents these employment effects, it abolishes much of the incentive for prices to return rapidly to their trend. PPP oriented exchange rate rules thus promote price level instability.

CHAPTER 6. HOW NECESSARY IS THE ASYMMETRIC MODEL? A FRACTIONAL COINTEGRATION TEST COMPARISON

Introduction

In chapter 5, we claimed that we found very positive evidence in favor of PPP due to two reasons: (1) a special-feature equal-weight data set which is more appropriate to test for PPP. and (2) the application of the asymmetric test methodology based on the awareness of the fact that the adjustment mechanism of the price index displays significant asymmetry. The gain due to second reason is the enhanced power of the asymmetric test methodology to detect asymmetric convergence. Now the question is: how much of the improvement can be attributed to this asymmetric test versus a better data set, if the latter plays a much more important role, then the focus of this paper on asymmetry would lose foundation. For example, what if we combine other more powerful test, such as fractional cointegration test, with this new data set? If the results turn out to support PPP enough, then the true scenario could be symmetric slow mean-reversion process rather than asymmetric threshold process. The fractional cointegration test based on spectral analysis can detect mean reversion more powerfully in that it allows for fractional integration, which exhibits reversion to a mean, but at a much slower rate than a stationary series. In the subsequent sections in this chapter, we'll give a description of the fractional cointegration as well as the detailed test procedure. Of course, the test results are given and some conclusions are made based on that.

Fractional Integration and Fractional Cointegration

Fractional Integartion

A fractional integrated process allows the series to evolve according to:

$$(6.1) \qquad \Phi(L)(1-L)^{d}X_t = \psi(L)\epsilon_t$$

where $\Phi(L)$ and $\psi(L)$ are polynomial lag operators with roots outside the unit circle and ϵ_t is white noise. If $0<d<1$, then we say the series X_t is fractional integrated of order d. It is worth noting that the fractional integrated series exhibit reversion to a mean, but at a much slower rate than a stationary series. Cheung and Lai (1993b) find that the tests tend to have more power than augmented Dickey-Fuller tests in detecting mean reversion, particularly for $0.35<d<0.65$. Because many people claim that the convergence of long run PPP tends to be very slow, this rather new fractional integration test and the correlated fractional cointegration test seem to be very suitful for testing PPP.

A Test for Fractional Cointegration

The hypothesis of fractional cointegration raises the problem of testing for fractional integration. Diebold and Rudebusch (1991) and Sowell (1990) observed that standard unit-root tests such as the Dickey-Fuller test may have low power against fractional alternatives. In our paper, a spectral regression-based test due to Geweke and Porter-Hudak (1983) is used to detect fractional integration in the equilibrium error μ_t, where μ_t is the least square residual of the cointegrating regression as implied by PPP. The detailed test procedure is as follows:

Step 1) Estimate equation (6.2) as the cointegrating regression

$$(6.2) \quad e_t = \alpha + \beta p_t + \mu_t$$

where e_t is the log of the exchange rate, p_t is the log of the price ratio, and μ_t is the residuals.

Step 2) Compute $G_t = (1-L)\mu_t$, define the sample size of $\{G\}$ as T, compute Fourier frequencies $\omega_j = 2\pi j/T$ (j=0,.... T-1).

Step 3) Estimate equation (6.3) as follows

$$(6.3) \quad \ln(I(\omega_j)) = \beta_0 + \beta_1 \ln(4\sin^2(\omega_j/2)) + \epsilon_t$$

where j = 1,2,....n. n = g(T), is an increasing function of T. and $I(\omega_j)$ is the periodogram at ordinate j, which is defined as (6.4):

$$(6.4) \quad I(\omega_j) = (|\sum_{1}^{T} G_t[(\cos(t\omega_j) - i\sin(t\omega_j))]|^2)/T$$

Under some regularity conditions on g (T), satisfied by, for example, T^{μ} for $0<\mu<1$, Geweke and Porter-Hudak (1983) showed that the least squares estimate of β_1 provides a consistent estimate of (1-d) and hypothesis testing concerning the value of d can be based on the t statistic of the regression coefficient. Note that this result only holds for low-frequency range, that's why we need to introduce a transformation function $n = g(T) = T^{\mu}$, where $0<\mu<1$. Easy to see, $n< T$.

In testing for fractional cointegration, however, the critical values for the GPH test are nonstandard, and those derived from the standard distribution cannot be used directly to evaluate the GPH estimate of d. This is because μ_t is not actually observed but obtained from minimizing the residual variance of the cointegrating regression, and the residual series thus obtained tends to bias toward being stationary. The null hypothesis of no cointegration is therefore expected to be rejected more often than suggested by the normal size of the GPH test. To cope with the problem, the empirical size of the GPH test in finite samples can be obtained using the simulation approach.

Cheung and Lai (1993b) used the Monte Carlo method in 50,000 replications to get the critical values for GPH test corresponding to a sample size T=76. A range of values of μ was used for the sample-size function $n = T^{\mu}$. They report results for μ =.55,.575, and.60. Table 6.1 reproduce the empirical distribution of the GPH test for cointegration.

Table 6.1: The Empirical Size of the GPH Test for Cointegration[a]

Percentile(%)	μ =.55	μ =.575	μ =.60
1	-2.886	-2.9	-2.879
5	-1.954	-1.955	-1.964
10	-1.515	-1.52	-1.531

[a]The sample size for the GPH spectral regression is given by $n = T^{\mu}$ where T =76. The empirical size is obtained based on 50,000 replications in simulation, assuming that the true system is of two noncointegrated random walk processes.

The Fractional Cointegration Test for PPP

Table 6.2 is the result of the GPH test for Purchasing Power Parity.

The implication is very straightforward: fractional cointegration test finds PPP convergence in 7 out of 15 pairs of countries at 5% significance level. This number is exactly as same as the number in Engle-Granger test. i.e., Fractional cointegration test does not provide more evidence in favor of PPP than conventional test. Therefore we can conclude that the real issue of conventional test is unawareness of the asymmetric adjustment mechanism rather than lack of power to detect very slow mean reversion.

Table 6.2:Results of the GPH Test for Fractional Cointegration[a]

Country Pair	μ =.55 d	μ =.55 H_0: d =1	μ =.575 d	μ =.575 H_0: d =1	μ =.60 d	μ =.60 H_0: d =1
CUS	-0.004	-5.676*	0.04	-5.958*	0.073	-6.032*
CF	0.594	-2.638*	0.607	-2.882*	0.658	-2.626*
CI	0.756	-0.908	0.683	-1.288	0.735	-1.18
CUK	0.634	-1.528	0.583	-1.93	0.627	-1.893
USF	0.651	-1.434	0.807	-0.755	0.876	-0.522
USI	0.102	-3.510*	0.123	-3.870*	0.206	-3.680*
USUK	0.268	-2.118*	0.483	-1.44	0.499	-1.559
DMUS	0.786	-1.304	0.787	-1.47	0.771	-1.759
DMC	0.301	-1.564	0.269	-1.849	0.413	-1.562
DMF	0.601	-1.514	0.673	-1.356	0.663	-1.564
DMI	-0.027	-5.127*	-0.03	-5.833*	0.008	-6.134*
DMUK	0.691	-2.312*	0.714	-2.400*	0.69	-2.848*
FI	-0.114	-6.847*	-0.084	-7.426*	0	-6.641*
FUK	0.505	-2.090*	0.639	-1.504	0.674	-1.5
IUK	0.655	-2.021*	0.617	-2.482*	0.592	-2.921*

[a]The sample size for the GPH test is given by $n = T^{\mu}$, where T = 67. Significance is indicated by * at the 5% level. The hypothesis H_0: d=1 is tested against the one-sided alternative of d<1. Critical values are based on simulated values given in Table 6.1.

CHAPTER 7. MORE GENERAL OR MORE SPECIFIC? A FOURIER CONVERGENCE TEST COMPARISON

Fourier Approximation of the Non-Linear Coefficients

In a very recent paper, Enders and Ludlow (1998) proposed a more general unit-root test and cointegration test which do not have to specify the functional form of the alternative hypothesis. This is in contrast to the linear specification in conventional tests and threshold specification in TAR and M-TAR tests. The point is often times we have no idea about how the convergence actually occurs. i.e., generally there is no reason to claim one specification is more preferred to another. By stating that a first-order Fourier approximation can capture many features of non-linear convergence, Enders & Ludlow (1998) deduced a couple of sets of critical values via Monte-Carlo simulation which can test the null hypothesis of no convergence versus the general alternative hypothesis of convergence. They showed that the new test statistics have substantially more power than standard tests. To see it more clearly, let's consider a simple modification of the AR(1) model in which the autoregressive coefficient is a time-dependent function denoted by $\alpha(t)$:

$$(7.1) \quad y_t = \alpha(t)y_{t-1} + \epsilon_t$$

where ϵ_t is a white noise disturbance with constant variance and $\alpha(t)$ is a deterministic funtion of t.

Under very weak conditions, the behavior of $\alpha(t)$ can be exactly represented by a sufficiently long Fourier series. For any desired level of accuracy, it is possible to write:

$$(7.2)\qquad \alpha(t)=\alpha_0+\sum_{k=1}^{n}[a_k\sin\frac{2\pi k}{T}\cdot t+b_k\cos\frac{2\pi k}{T}\cdot t]+e_n(t)$$

where n refers to the number of summands contained in the approximation of $\alpha(t)$ and $e_n(t)$ is the approximation error.

In order to keep the problem tractable, we consider only a Fourier approximation for the case of n=1 and k is any integer in the interval 1 to T/2. Hence, equation (7.2) reduces to:

$$(7.3)\qquad \alpha(t)=\alpha_0+a_1\sin\frac{2\pi k}{T}\cdot t+b_1\cos\frac{2\pi k}{T}\cdot t$$

Enders & Ludlow (1998) showed that the convergence of $\{y_t\}$ depends on the relationship between α_0, a_1 and b_1. It has nothing to do with the frequency k. Specifically, the necessary and sufficient conditions for the convergence of the $\{y_t\}$ sequence is $c< r^2/4$ and $r < 2$. where:

$$(7.4)\qquad \begin{aligned} c&=\alpha_0-1\\ r&=\sqrt{a_1^2+b_1^2}\end{aligned}$$

Fourier Convergence Test of Purchasing Power Parity

To test for PPP, perform the following procedures:

Step 1) regress the log real exchange rate on a constant and save the residuals or demeaned series as $\{r_t\}$.

Step 2) In order to select the most appropriate frequency k, estimate the following regression using each interger value of k in the interval 1 to T/2:

$$(7.5) \quad \Delta r(t) = [c + a_1 \sin\frac{2\pi k}{T}\cdot t + b_1 \cos\frac{2\pi k}{T}\cdot t] r_{t-1} + \epsilon_t$$

The value of k resulting in the smallest residual sum of squares is called k* and the coefficients associated with that frequency are called c*, a_1* and b_1*. The t-statistic for the null hypothesis c*=0 was recorded along with the F-statistics for three different null hypotheses regarding various combinations of the coefficients. The F_all statistic tests the null hypothesis c*= a_1* = b_1* = 0. Similarly, the F_trig statistic tests the null hypothesis a_1* = b_1* = 0 and the t-test for the restriction c* = 0 is called the c* statistic. The value r* was calculated as $[(a_1*)^2 + (b_1*)^2]^{1/2}$ and the test statistic for the non-linear restriction c* = (r*) /4 is called cr*. The distributions for these four statistics are reported by Enders and Ludlow (1998) for sample sizes of 50. 100 and. They are reproduced in Table 7.1.

Table 7.1: Unit Root Tests

	The F-all$_\mu$ Statistic			The F-trig$_\mu$ Statistic		
#	0.9	0.95	0.99	0.9	0.95	0.99
50	5.79	6.59	8.65	5.94	6.81	8.91
100	5.98	6.72	8.5	6.42	7.24	9.1
	The c*$_\mu$ Statistic			**The cr*$_\mu$ Satistic**		
#	0.9	0.95	0.99	0.9	0.95	0.99
50	-2.65	-3.07	-3.89	8.33	10.81	16.67
100	-2.6	-3	-3.79	7.68	9.92	15.32

Interpretation of the Test Results

The test results are reported in Table 7.2. Suppose we accept PPP only when both of the F_all statistic and cr* statistic are significant, then we can conclude that at 5% level. PPP is valid in 8 out of 15 pairs of countries. Comparing this with the fact that Dickey-Fuller test found the validity of PPP in 9 out of 15 pairs, we claim that Fourier convergence test does not perform better than conventional test in supporting PPP. However, this does not mean Fourier convergence test is useless, all it implies is that the Fourier convergence test might be too general in this fitting, given the theoretical awareness of the asymmetry of the price index. In other words, if we take into account the obvious dominant performance of the threshold model test over Dickey-Fuller test, we may realize that the threshold model does a much better job on catching the true adjustment mechanism than both of Dickey-Fuller test and Fourier convergence test.

So there is no definite answer to the issue regarding whether the more general test or more specific test works better. It depends. If the specific one catches the true mechanism correctly, it sure will overpowers the general one. If the true scenario is misspecified, then the specific one will be bad. Therefore the good suggestion is: if we do have prior information about what is going on, specific model is preferred. If we don't have valuable prior information, use general approach, that is why Enders and Ludlow (1998) proposed the Fourier convergence test.

Table 7.2: Fourier Convergence Tests[a]

CUS

Dickey-Fuller Test					Fourier Convergence			
Lags	1	2	3		Lags	1	2	3
ρ	-0.235	-0.24	-0.276		c*	-0.212	-0.210	-0.243
	(-3.01**)	(-2.84)	(-3.04)			(-2.90*)	(-2.61)	(-2.84)
					F_all	6.88**	6.42	6.85
					F_trig	5.1	4.98	4.95
					cr*	10.29*	8.46	9.72
AIC	-151.2	-149.3	-148.5		AIC	-159	-157	-156.4
SBC	-144.8	-140.7	-137.8		SBC	-150.5	-146.3	-143.5
D-W	1.99	1.98	1.98		D-W	2	2.01	2
Q(2)	0.96	0.98	0.94		Q(2)	0.98	0.98	0.93
Q(4)	0.89	0.9	0.98		Q(4)	0.85	0.85	0.89
Q(8)	0.94	0.95	0.98		Q(8)	0.64	0.64	0.

CF

Dickey-Fuller Test					Fourier Convergence			
Lags	1	2	3		Lags	1	2	3
ρ	-0.212	-0.285	-0.301		c*	-0.165	-0.234	-0.249
	(-2.56)	(-3.42**)	(-3.29)			(-2.16)	(-3.00*)	(-2.91)
					F_all	7.17	8.81***	8.41
					F_trig	6.93	6.35*	6.23
					cr*	6.52	11.31**	10.53
AIC	18.99	13.83	15.62		AIC	8.27	3.76	5.55
SBC	25.42	22.4	26.34		SBC	16.84	14.48	18.41
D-W	2.05	2.02	1.96		D-W	2.06	1.97	1.92
Q(2)	0.11	0.9	0.87		Q(2)	0.22	0.98	0.93
Q(4)	0.19	0.83	0.92		Q(4)	0.38	0.97	0.99
Q(8)	0.33	0.85	0.87		Q(8)	0.08	0.45	0.43

[a] * indicates 10% significance level, ** indicates 5% significance level, *** indicates 1% significance level.

Table 7.2: (continued)

CI

Dickey-Fuller Test					Fourier Convergence			
Lags	1	2	3		Lags	1	2	3
ρ	-0.251	-0.348	-0.344		c*	-0.075	-0.152	-0.07
	(-2.70)	(-3.88^{***})	(-3.41)			(-1.03)	(-2.01)	(-0.81)
					F_all	21.35	22.58	23.13^{***}
					F_trig	25.53	21.2	24.2^{***}
					cr*	6.6	11.56	4.81
AIC	51.54	41.76	43.75		AIC	14.65	9.43	7.27
SBC	57.97	50.33	54.46		SBC	23.22	20.15	20.13
D-W	1.92	1.98	2		D-W	2	1.79	1.91
Q(2)	0.04	0.97	0.96		Q(2)	0.62	0.41	0.4
Q(4)	0.07	0.97	0.98		Q(4)	0.42	0.43	0.75
Q(8)	0.11	0.78	0.78		Q(8)	0.42	0.27	0.27

CUK

Dickey-Fuller Test					Fourier Convergence			
Lags	1	2	3		Lags	1	2	3
ρ	-0.074	-0.093	-0.072		c*	-0.057	-0.074	-0.066
	(-1.50)	(-1.80)	(-1.34)			(-1.16)	(-1.49)	(-1.26)
					F_all	3.69	4.44	3.41
					F_trig	4.28	4.78	4.06
					cr*	1.87	2.95	2.07
AIC	-43.38	-42.97	-42.69		AIC	-49.43	-50.23	-48.62
SBC	-36.95	-34.39	-31.98		SBC	-40.86	-39.51	-35.76
D-W	1.99	1.87	1.94		D-W	1.92	1.83	1.87
Q(2)	0.44	0.96	1		Q(2)	0.44	0.94	0.96
Q(4)	0.62	0.88	1		Q(4)	0.67	0.95	0.99
Q(8)	0.86	0.9	0.99		Q(8)	0.89	0.94	0.97

Table 7.2: (continued)

USF

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.229	-0.299	-0.315	c*	-0.186	-0.255	-0.275
	(-2.76)	(-3.54**)	(-3.38)		(-2.39)	(-3.21**)	(-3.16)
				F_all	6.53	8.48**	8.09
				F_trig	5.5	5.51	5.49
				cr*	7.67	12.8**	12.14
AIC	12.26	8.11	9.92	AIC	3.91	-0.57	1.07
SBC	18.69	16.68	20.63	SBC	12.48	10.15	13.93
D-W	2.05	2.03	1.98	D-W	2.06	2.03	1.97
Q(2)	0.15	0.96	0.94	Q(2)	0.22	0.96	0.95
Q(4)	0.31	0.91	0.96	Q(4)	0.36	0.97	0.99
Q(8)	0.4	0.92	0.93	Q(8)	0.1	0.54	0.54

USI

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.302	-0.416	-0.417	c*	-0.099	-0.198	-0.103
	(-2.98)	(-4.20***)	(-3.70)		(-1.25)	(-2.51)	(-1.15)
				F_all	23.12	27.91	28.43***
				F_trig	26.55	25.7	29.1***
				cr*	9.09	17.53	8.13
AIC	44.65	35.25	37.25	AIC	6.52	-2.54	-4.9
SBC	51.08	43.82	47.96	SBC	15.09	8.18	7.96
D-W	1.92	2.01	2.01	D-W	1.89	1.68	1.75
Q(2)	0.08	0.99	0.996	Q(2)	0.08	0.62	0.87
Q(4)	0.09	0.99	0.99	Q(4)	0.07	0.61	0.99
Q(8)	0.11	0.86	0.86	Q(8)	0.07	0.58	0.85

Table 7.2: (continued)

USUK

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.096	-0.091	-0.086	c*	-0.033	-0.025	-0.024
	(-1.91)	(-1.70)	(-1.53)		(-0.57)	(-0.43)	(-0.40)
				F_all	6.03^{*}	5.94	5.65
				F_trig	6.84^{**}	7.11	6.98
				cr*	0.91	0.68	0.62
AIC	-48.32	-46.41	-44.49	AIC	-59.23	-57.98	-56.02
SBC	-41.89	-37.84	-33.77	SBC	-50.66	-47.27	-43.16
D-W	1.94	1.96	1.96	D-W	1.89	1.91	1.9
Q(2)	0.99	0.96	0.98	Q(2)	0.71	0.98	0.99
Q(4)	0.9999	0.999	0.999	Q(4)	0.87	0.98	0.98
Q(8)	0.92	0.92	0.92	Q(8)	0.96	0.99	0.99

DMUS

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.103	-0.097	-0.0999	c*	-0.098	-0.093	-0.098
	(-2.31)	(-2.06)	(-2.05)		(-2.43)	(-2.18)	(-2.22)
				F_all	6.83^{**}	6.34	6.31
				F_trig	7.06^{**}	6.94	6.93
				cr*	$-\infty^{***}$	$-\infty$	$-\infty$
AIC	-49.42	-47.61	-45.68	AIC	-60.69	-58.85	-57.12
SBC	-42.99	-39.03	-34.96	SBC	-52.12	-48.13	-44.27
D-W	1.94	1.98	1.99	D-W	2.03	2.06	2.08
Q(2)	0.89	0.997	0.998	Q(2)	0.82	0.92	0.94
Q(4)	0.78	0.83	0.86	Q(4)	0.82	0.88	0.83
Q(8)	0.95	0.98	0.98	Q(8)	0.95	0.97	0.94

Table 7.2: (continued)

DMC

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.088	-0.094	-0.081	c*	-0.082	-0.089	-0.081
	(-1.999)	(-2.04)	(-1.70)		(-1.96)	(-2.04)	(-1.80)
				F_all	4.51	4.59	3.86
				F_trig	4.56	4.56	4.17
				cr*	$-\infty^{***}$	$-\infty$	$-\infty$
AIC	-43.47	-41.71	-40.89	AIC	-50.1	-48.51	-47.06
SBC	-37.04	-33.13	-30.18	SBC	-41.53	-37.79	-34.2
D-W	2.02	1.96	1.94	D-W	2.1	2.03	2.01
Q(2)	0.79	0.98	0.95	Q(2)	0.83	0.96	0.86
Q(4)	0.54	0.63	0.64	Q(4)	0.52	0.57	0.65
Q(8)	0.61	0.61	0.76	Q(8)	0.7	0.7	0.82

DMF

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.158	-0.206	-0.256	c*	-0.295	-0.332	-0.382
	(-2.09)	(-2.75)	(-3.29**)		(-3.18)	(-3.61)	(-4.30***)
				F_all	14.33	14.86	18.71***
				F_trig	18.05	16.48	19.15***
				cr*	15.69	18.79	25.86***
AIC	-1.46	-5.7	-7.63	AIC	-29.52	-32.04	-38.01
SBC	4.96	2.88	3.09	SBC	-20.95	-21.32	-25.15
D-W	1.95	2.15	1.93	D-W	1.82	2.07	1.91
Q(2)	0.08	0.72	0.74	Q(2)	0.16	0.74	0.98
Q(4)	0.1	0.18	0.86	Q(4)	0.29	0.26	0.91
Q(8)	0.13	0.42	0.87	Q(8)	0.62	0.53	0.97

Table 7.2: (continued)

DMI

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.233	-0.318	-0.334	c*	-0.102	-0.189	-0.200
	(-2.40)	(-3.34**)	(-3.20)		(-1.18)	(-2.30)	(-2.21)
				F_all	10.96	15.18***	14.53
				F_trig	12.42	14.54***	14.24
				cr*	4.89	11.51**	10.27
AIC	39.74	32.52	34.34	AIC	19.61	8.91	10.82
SBC	46.17	41.09	45.06	SBC	28.18	19.63	23.67
D-W	1.84	2.03	1.98	D-W	2.13	2.34	2.3
Q(2)	0.08	0.83	0.89	Q(2)	0.06	0.23	0.31
Q(4)	0.08	0.84	0.89	Q(4)	0.05	0.05	0.09
Q(8)	0.17	0.88	0.91	Q(8)	0.09	0.02	0.03

DMUK

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.120	-0.098	-0.120	c*	-0.091	-0.087	-0.104
	(-2.31)	(-1.82)	(-2.21)		(-1.85)	(-1.70)	(-1.93)
				F_all	5.35	3.8	3.59
				F_trig	4.97	3.86	2.78
				cr*	4.47	3.62	4.43
AIC	-81.88	-81.9	-83.02	AIC	-89.66	-87.75	-86.84
SBC	-75.45	-73.33	-72.31	SBC	-81.09	-77.03	-73.99
D-W	1.86	1.9	2.01	D-W	1.91	1.93	1.94
Q(2)	0.27	0.93	0.95	Q(2)	0.92	0.98	0.98
Q(4)	0.38	0.76	0.99	Q(4)	0.9	0.9	0.98
Q(8)	0.6	0.9	0.8	Q(8)	0.74	0.75	0.72

Table 7.2: (continued)

FI

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.185	-0.202	-0.200	c*	-0.112	-0.098	-0.108
	(-2.34)	(-2.43)	(-2.29)		(-1.66)	(-1.35)	(-1.42)
				F_all	11.36***	11.34	10.99
				F_trig	13.15***	12.81	12.76
				cr*	6.79	5.51	5.61
AIC	0.74	2.22	4.22	AIC	-20.47	-18.82	-17.08
SBC	7.17	10.79	14.93	SBC	-11.9	-8.11	-4.23
D-W	1.99	1.99	1.99	D-W	2.05	2.06	2.07
Q(2)	0.83	0.998	0.998	Q(2)	0.79	0.95	0.93
Q(4)	0.95	0.98	0.99	Q(4)	0.85	0.94	0.93
Q(8)	0.96	0.98	0.98	Q(8)	0.47	0.48	0.48

FUK

Dickey-Fuller Test				Fourier Convergence			
Lags	1	2	3	Lags	1	2	3
ρ	-0.296	-0.377	-0.467	c*	-0.185	-0.247	-0.347
	(-3.01)	(-3.76)	(-4.36***)		(-2.11)	(-2.67)	(-3.69**)
				F_all	11.37	12.19	16.43***
				F_trig	11.05	9.25	11.67***
				cr*	10.36	13.48	22.44***
AIC	-1.85	-5.81	-8.22	AIC	-19.71	-21.05	-27.59
SBC	4.58	2.76	2.49	SBC	-11.14	-10.33	-14.73
D-W	1.99	2.14	1.99	D-W	1.91	2.05	1.92
Q(2)	0.21	0.81	0.997	Q(2)	0.48	0.9	0.99
Q(4)	0.31	0.53	0.97	Q(4)	0.5	0.49	0.62
Q(8)	0.24	0.75	0.998	Q(8)	0.2	0.31	0.67

Table 7.2: (continued)

IUK

Dickey-Fuller Test					Fourier Convergence			
Lags	1	2	3		Lags	1	2	3
ρ	-0.235	-0.304	-0.287		c*	-0.070	-0.132	-0.096
	(-2.44)	(-3.16**)	(-2.75)			(-0.87)	(-1.62)	(-1.10)
					F_all	15.96	17.65***	16.75
					F_trig	19.16	18.5***	18.98
					cr*	4.69	8.14	5.08
AIC	38.25	33.82	35.59		AIC	8.76	4.78	5.48
SBC	44.68	42.39	46.31		SBC	17.33	15.49	18.34
D-W	1.84	1.93	1.99		D-W	2.07	2.03	2.16
Q(2)	0.2	0.88	0.84		Q(2)	0.12	0.67	0.41
Q(4)	0.12	0.89	0.87		Q(4)	0.1	0.63	0.65
Q(8)	0.43	0.84	0.87		Q(8)	0.08	0.09	0.11

CHAPTER 8. SUMMARY

This dissertation focuses on testing purchasing power parity. The special feature of the paper is that we jump from linear to nonlinear approach. Theoretical progress recently made by Enders & Granger (1998), Enders & Siklos (1998) and Enders & Ludlow (1998) triggered our research on this topic. The empirical results are quite consistent to our theoretical expectation or can be reasonably explained. It is worth noting that our RATS results are very supportive to the validity of PPP, which is in very contrast to the previous empirical finding.

However, we cannot give all credits to the application of this nonlinear approach. As a matter of fact, another special characteristic of this dissertation is that all through the paper, we use a very nice constructed data set provided by John Pippenger. We argued in chapter five that this equal-weight data set is particularly appropriate to testing for PPP.

So the benefit comes from two perspectives: a better model which captures the asymmetric adjustment mechanism of price index and a better data set. We know both of them contribute to the empirical success, but since we spend so much time on TAR model and M-TAR model, it is to our great interest to make sure the contribution of the better model is not trivial. Toward this end, a fractional cointegration test is applied to the same data set in chapter six for a comparison purpose. We found that the more powerful slow-mean-reversion detecting method did not work better than Engle-Granger test. Therefore we claim that the attention on the asymmetric test methodology should be given.

Another interesting comparison as we did in chapter seven is the comparison between a very general Fourier convergence test and more specific threshold model test, It seems that in this case, the general model is not very much preferred to conventional test, let alone the threshold model. The basic sense behind this is if we do have theoretical prior information regarding the real struture of the model, like what we carefully reviewed in chapter three, a correct specific model is the best choice. On the other hand, if we don't have valuable information about what is going on, like often times happened in the real world, a more general approximation could be better, it could be a second best in contrast to the real model, but it surely will be much better than a wrongly specified model.

REFERENCES

Abuaf, N. and P. Jorion. (1990). "Purchasing Power Parity in the Long Run". *Journal of Finance*, 45: 157-174.

Adler, M. and B. Lehman. (1983). "Deviations from Purchasing Power Parity in the Long Run". *Journal of Finance*, 39:1471-1487.

Ardeni, Pier G. and Diego Lubian. (1989). "Purchasing Power Parity During the 1920s". *Economic Letters*, 30: 257-62.

Balassa,B. (1964). "The Purchasing Power Parity Doctrine: A Reappraisal". *Journal of Political Economy*, 72: 584-596.

Balke,Nathan S. and Thomas B.Fomby. (1992). "Threshold Cointegration" *Southern Methodist University Working Paper*, mimeo. May,1992.

Ball, L. and Mankiw, N.G. (1994). "Asymmetric Price Adjustment and Economic Fluctuations". *The Economic Journal*, March,1994. 247-261.

Ball, L. , Mankiw, N.G. and Romer, D. (1988). "The New Keynesian Economics and the Output-Inflation Tradeoff". *Brookings Papers on Economic Activity*, no.1, pp.1-65.

Barro, R. (1984). *Macroeconomics* , John Wiley, New York.

Borenstein, S. , A.Colin Cameron and Richard Gilbert. (1997) "Do Gasoline Prices Respond Asymmetrically to Crude Oil Price Changes?". *Quarterly Journal of Economics*, Feb, 305-339.

Brainard, W. (1967) "Uncertainty and the Effectiveness of Policy". *American Economic Review*, May 1967, 411-25.

Breuer, J. (1994). "Purchasing Power Parity : A Survey of and Challenge to Recent Literature". in J.Williamson, ed., *Estimating Equilibrium Exchange Rates* , Institute for International Economics, Washington, DC.

Caplin,A.S. and Spulber, D.F. (1987). "Menu Costs and the Neutrality of Money". *Quarterly Journal of Economics*, 106: 683-708.

Cassel,G.(1922). *Money and Foreign Exchange after 1914*, Macmillan, New York.

Cheung,Yin-Wong. and Kon S.Lai. (1993a). "Long-Run Purchasing Power Parity during the Recent Float". *Journal of International Economics*, 34: 181-92.

Cheung,Yin-Wong. and Kon S.Lai. (1993b). "A Fractional Cointegration Analysis of Purchasing Power Parity". *Journal of Business and Economic Statistics*, 11: 103-12.

Chordhury,A.and F.Sdogati.(1993). "Purchasing Power Parity in the Major EMS Countries :the Role of Price and Exchange Rate Adjustment". *Journal of Macroeconomics*, 15:25-45.

Choudry,Taufiq, Robert McNowm, and Myles Wallace. (1991)."Purchasing Power Parity and the Canadian Float in the 1950s". *Review of Economics and Statistics*, 73:558-63.

Cochrane,John. (1988). "How Big is the Random Walk in GNP?". *Journal of Political Economy*, 96: 893-920.

Corbae,D.and S.Ouliaris.(1988). "Cointegration and Tests of Purchasing Power Parity". *Review of Economics and Statistics*, 70:508-21.

Cumby,R.(1993). "Forecasting Exchange Rates on the Hamburger Standard: What You See is What You Get with McParity". mimeo, Stern School of Business, NYU.

Darby,M.(1983). "Movements in Purchasing Power Parity: The Short and Long Runs", in: M.Darby and J.Lothian,eds.,*The International Transmission of Inflation*, University of Chicago Press,Chicago.

Davuytan,Nurham and John Pippenger. (1985). "Purchasing Power Parity Did Not Collapse During the 1970s". *American Economic Review*, 75: 1151-58.

Dickey,David and Wayne A.Fuller. (1979)."Distribution of the Estimates for Autoregressive Time Series with a Unit Root."*Journal of the American Statistical Association*, 74 427-431.

Dickey, David and Wayne A.Fuller. (1981). "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root". *Econometrica*, 49: 1057-72.

Diebold,Francis X. and Rudebusch, G.D. (1991). "On the Power of Dickey-Fuller Tests Against Fractional Alternatives". *Economics Letters*, 35, 155-160.

Diebold,Francis X., Steven Husted, and Mark Rush. (1991). "Real Exchange Rates under the Gold Standard". *Journal of Political Economy*, 99:1252-71.

Dornbusch,R. (1976). "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, 84:1161-76.

Dornbusch,R. (1980). *Open Economy Macroeconomics*, Basic Books, Inc. Publishers, New York.

Dornbusch, R. (1982). "PPP Exchange-Rate Rules and Macroeconomic Stability". *Journal of Political Economy*, 90:159-165

Edison, H.J. (1987). "Purchasing Power Parity in the Long Run" *Journal of Money, Credit, and Banking* 19, no. 3, 376-387.

Edison, H.J. and J.T. Klovland (1987). "A Quantitative Reassessment of the Purchasing Power Parirty Hypothesis". *Journal of Applied Econometrics*, 2: 309-333.

Enders, Walter (1988). "ARIMA and Cointegration Tests of PPP Under Fixed and Flexible Exchange Rate Regimes". *Review of Economics and Statistics*, 70 : 504-08.

Enders, Walter and C.W.J.Granger. (1998). "Unit-Root Tests and Asymmetric Adjustment with an Example Using the Term Structure of Interest Rates". *Journal of Business and Economic Statistics*, July 1998, 304-311.

Enders, Walter and Ludlow, J (1998). "Non-Linear Convergence: Tests for an Attractor Using a Fourier Approximation". mimeo. Iowa State University Working Paper, Ames, IA.

Enders, Walter and Pierre L. Siklos.(1998). "The Term Structure: Testing for an Equilibrium with Asymmetric Adjustment". mimeo. Iowa State University Working Paper, Ames, IA.

Engle,R.F. and B.Yoo (1987). "Forecasting and Testing in Cointegrated Systems." *Journal of Econometrics*,35.

Engle, R.F. and C.W.J.Granger (1987). " Cointegration and Error Correction: Representation, Estimation, and Testing". *Econometrica*, 55: 251-276.

Engle,C. and J.Rogers.(1994). "How Wide is the Border?" NBER working paper, No.4829

Fisher, E and J.Park. (1991). "Testing Purchasing Power Parity Under the Null Hypothesis of Cointegration". *The Economic Journal*, 101: 1476-1484.

Flynn, N. Alston. and Janice L. Boucher. (1993). "Tests of Long Run Purchasing Power Parity using Alternative Methodologies". *Journal of Macroeconomics*, 15 : 109-22.

Frankel, J. (1986). "International Capital Mobility and Crowding Out in the U.S. Economy: Imperfect Integration of Financial Markets or of Goods Markets?" in R.Hafer, ed., *How Open is the U.S. Economy?* , Lexington Books, Lexington, MA.

Frankel, J. (1990)."Zen and the Art of Modern Macroeconomics". in W.Haraf and T.Willett. eds., *Monetary Policy for a Volatile Global Economy*, AEI, Washington, DC.

Frenkel, J. (1978). "Quantifying International Capital Mobility in the 1980s" , in: D.Bernheim and J.Shoven, eds., *National Saving and Economic Performance*. University of Chicago Press,Chicago. 227-260.

Frenkel, J.A. and Johnson, H.G. (1978). *The Monetary Approach to the Balance of Payments*. George Allen and Unwin, London.

Froot, K.A. and Rogoff, K. (1995). "Respectives on PPP and Long-Run Real Exchange Rates". *Handbook of International Economics*, Vol 3 , G.Grossman & K.Rogoff, Elsevier Science Pub.Co, New York, USA.

Geweke, J. and Porter-Hudak,S. (1983). "The Estimation and Application of Long Memory Time Series Models". *Journal of Time Series Analysis*, 4: 221-238.

Glen, J. (1992). "Real Exchange Rates in the Short, Medium, and Long Run". *Journal of International Economics*,33:147-66.

Giovannetti, G. (1992). "A Survey of Recent Empirical Tests of the Purchasing Power Parity Hypothesis".*Banca Nazionale del Lavoro Quarterly Review*, 180:81-101.

Granger, C.W.J. and T.H.Lee. (1989). "Investigation of Production, Sales, and Inventory Relationships using Multicointegration and Nonsymmetric Error-Correction Models". *Journal of Applied Econometrics*,4: S145-S159.

Green, E. and R. Porter. (1984). "Non-cooperative Collusion Under Imperfect Price Information". *Econometrica*, LII : 87-100.

Greenwald, B. and J.Stiglitz. (1989). "Toward a Theory of Rigidities". *American Economic Review* , May 79, 364-9.

Grilli, Vittorio and Graciela Kaminsky. (1991). "Nominal Exchange Rate Regimes and the Real Exchange Rate : Evidence from the United States and Great Britain, 1885-1986". *Journal of Monetary Economics*, 27: 191-212.

Hakkio, C. (1984)."A Reexamination of Purchasing Power Parity". *Journal of International Economics* 17: 265-277.

Huizinga, J. (1987). "An Empirical Investigation of the Long-Run Behavior of Real Exchange Rates". in: K.Brunner and A.Meltzer, eds., *Carnegie-Rochester Series on Public Policy* 27:149-215.

Johansen, Soren. (1988). "Statistical Analysis of Cointegrating Vectors". *Journal of Economic Dynamics and Control,* 12: 231-254.

Kim, H.(1990). "Purchasing Power Parity in the Long Run : A Cointegration Approach". *Journal of Money, Credit, and Banking,* 22: 491-503.

Knetter, M. (1989). "Price Discrimination by U.S. and German Exporters". *American Economic Review*, 79: 198-210.

Kravis, I., Heston, A. and Summers, R. (1978). "Real GDP Per Capita for More than One Hundred Countries". *Economic Journal*, 88, June, 215-42.

Kravis, I., Heston, A. and Summers, R. (1982). *World Product and Income: International Comparisons of Real Gross Product*, Baltimore, Johns Hopkins University Press.

Kravis, I. and Lipsey, R. (1983). "Toward an Explanation of National Price Levels". *Princeton Studies in International Finance* No.52, Princeton: Princeton University Press.

Kravis, I. and Lipsey, R. (1978). "Price Behavior in the Light of Balance of Payments Theories"*Journal of International Economics* 8(2), May, 193-246.

Krugman, Paul. (1987). "Pricing to Market When the Exchange Rate Changes". *Real-Financial Linkages among Open Economies*. edited by S.W. Arndt and J.D. Richardson, MIT Press,Cambridge, Massachusetts.

Kugler, P and C.Lenz. (1993). "Multivariate Cointegration Analysis and Long Run Validity of PPP". *Review of Economics and Statistics*,75: 180-184.

Liu, Peter C. (1993). "Purchasing Power Parity in Latin America : A Cointegration Analysis".*Weltwirtschaftliches Archiv* : 662-80.

Lothian, J. And M. Taylor. (1994). "Real Exchange Rate Behavior: The Recent Float from the Perspective of the Past Two Centuries". *Journal of Political Economy*, in press.

Mark, N.(1990) "Real Exchange Rates in the Long Run: An Empirical Investigation". *Journal of International Economics*, 28:115-136.

McNown, Robert and Myles S. Wallace. (1989). "National Price Levels, Purchasing Power Parity, and Cointegration : A Test of Four High Inflation Economies". *Journal of International Money and Finance* 8: 533-46.

Meese, R. and K.Rogoff. (1988). "Was it Real? The Exchange Rate Interest Differential Relation over the Modern Floating Exchange Rate Period". *Journal of Finance*. 43:933-948.

Neftci, Salih N.(1984). "Are Economic Time Series Asymmetric over the Business Cycle?". *Journal of Political Economy* 92. April, 1984. 307-28.

Patel, Jayendu.(1990). "Purchasing Power Parity as a Long Run Relation". *Journal of Applied Econometrics*.5: 367-79.

Perron, P.(1989). "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis". *Econometrica*, 57: 1361-1401.

Perron, P. and T.J.Vogelsang (1992). "Nonstationarity and Level Shifts with an Application to Purchasing Power Parity". *Journal of Business and Economic Statistics*,10: 301-20.

Phillips, P.(1987a) ."Time Series Regression with a Unit Root". *Econometrica*,55:297-332.

Phillips, P.(1987b). "Asymptotic Expansions in Nonstationary Vector Autoregressions". *Econometricic Theory* 3: 45-68.

Pippenger, Michael K. (1993). "Cointegration Tests of Purchasing Power Parity:The Case of Swiss Exchange Rates". *Journal of International Money and Finance*, 12: 46-61.

Potter, Simon.(1995). "A Nonlinear Approach to US GNP". *Journal of Applied Econometrics*, 10. (April-June 1995). 109-25

Rogers, J.H. and M.A. Jenkins. (1993). "Haircuts or Hysteresis? Sources of Movements in Real Exchange Rates". mimeo, Pennsylvania State University.

Samuelson, P.A. (1964). "Theoretical Notes on Trade Problems". *Review of Economics and Statistics* 46: 145-164.

Sichel, Daniel E. (1993). "Business Cycle Asymmetry: A Deeper Look". *Economic Inquiry* 31:224-36.

Sowell, F. (1990). "The Fractional Unit Root Distribution". *Econometrica*, 58: 495-505.

Taylor, J. (1979). "Staggered wage setting in a macro model". *American Economic Review*, papers and proceedings. 108-18.

Taylor, Mark and Patrick C. McMahon. (1988). "Long Run Purchasing Power Parity in the 1920s".*European Economic Review*,32 : 179-97.

Terasvirta,T and H.M.Anderson. (1992)."Characterizing Nonlinearities in Business Cycles using Smooth Transition Autoregressive Models." *Journal of Applied Econometrics*, 7:S119-S139.

Tong, Howell.(1983). *Threshold Models in Non-Linear Time Series Analysis*. Spring-Verlag, New York.

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IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6″

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